Robert C. Thompson Matrix Meeting 2007

249 Parker Hall, Auburn University, March 24

Abstracts

1. **Marina Arav** matmxa@langate.gsu.edu
Department of Mathematics & Statistics, Georgia State Univesity, Georgia, USA

   Title: Rational realizations of the minimum rank of a sign pattern matrix

   Abstract: A sign pattern matrix is a matrix whose entries come from the set +, -, 0. The minimum rank of a sign pattern matrix A is the minimum of the ranks of the real matrices whose entries have signs equal to the corresponding entries of A. It is conjectured that the minimum rank of every sign pattern matrix can be realized by a rational matrix. The equivalence of this conjecture to several seemingly unrelated statements is established. For some special cases, such as when A is entrywise nonzero, or the minimum rank of A is at most 2, or the minimum rank is least n-1 (where A is m by n), the conjecture is shown to hold. Connections between this conjecture and the existence of positive rational solutions of certain systems of homogeneous quadratic polynomial equations with each coefficient equal to either 1 or -1 are investigated.

   This is a joint work with Frank Hall, Selcuk Koyuncu, Zhongshan Li, and Bhaskara Rao.

2. **Jörg Feldvoss** jfeldvoss@jaguar1.usouthal.edu
Department of Mathematics & Statistics, University of South Alabama, Alabama, USA

   Title: Poisson superbrackets, the classical Yang-Baxter operator, and determinants

   Abstract: In this talk we will establish the classical Yang-Baxter operator of a Lie algebra $\mathfrak{a}$ as a quadratic mapping via Drinfeld's Poisson superbracket on the exterior algebra of $\mathfrak{a}$. Moreover, we show that (up to a constant factor) the classical Yang-Baxter operator of a three-dimensional simple Lie algebra $\mathfrak{g}$ is the composition of the determinant and a faithful two-dimensional representation applied to the Lie bracket of $\mathfrak{g}$.

3. **Jane M. Day** janemday@earthlink.net
Department of Mathematics, San Jose State University, California, USA

   Title: Application of a singular value inequality to graph energy

   Abstract: The energy $\mathcal{E}(G)$ of a graph G is the sum of the singular values of its adjacency matrix. If one edge is removed from G, or more generally all edges of an induced subgraph H, the energy can increase or decrease by at most $\mathcal{E}(H)$. When the

adjacency matrix of H is nonsingular, the lower bound will be attained if and only if no edges in G join H and $G-H$, and this is still true for some but not all singular H. The upper bound cannot be attained. The proof uses the equality case of a classic inequality for singular values of a matrix sum. It is a joint work with Wasin So.

4. **Peter M. Gibson**, gibson@math.uah.edu
Department of Mathematical Sciences, University of Alabama in Huntsville, Alabama, USA

   Title: On eigenvalues of certain classes of matrices

   Abstract: In this talk an elementary proof of the following is presented. (1) For a Hermitian matrix A of order $n \geq 2$, if each principal submatrix of order 2 is positive semidefinite then $|\lambda| \leq$ trace A for each eigenvalue λ of A . Let R_n be the positive real symmetric matrix of order n with (i,j) entry equal to $\binom{i+j-2}{j-1}/\binom{2n-2}{n-1}$, let x be a real number, and let $\mu_n(x)$ be the Perron root of the Hadamard (or entry wise) power $R_n^{(x)}$. Using (1) and earlier work by Ashrafi and the speaker it is shown that $\lim_{n\to\infty} \mu_n(x) = \frac{4^x}{4^x-1}$ for all $x > 0$. Related results are also discussed.

5. **Hongyu He**, hongyu@math.lsu.edu
Department of Mathematics, Louisiana State University, Louisiana, USA

   Title: Linear algebra method in reconstruction

   Abstract: I will discuss reconstruction of symmetric matrices. I will also give a proof of Tutte's theorem regarding the eigenvalues of hypomorphic matrices.

6. **Huajun Huang**, huanghu@auburn.edu
Department of Mathematics & Statistics, Auburn University, Alabama, USA

   Title: On Gelfand-Naimark decomposition of a nonsingular matrix

   Abstract: Let $\mathbb{F} = \mathbb{C}$ or $\mathbb{R}$ and $A \in \mathrm{GL}_n(\mathbb{F})$. Let $s(A) \in \mathbb{R}^n_+$ be the singular values of A, $\lambda(A) \in \mathbb{C}^n$ the unordered n-tuple of eigenvalues of A, $a(A) := \operatorname{diag} R \in \mathbb{R}^n_+$, where $A = QR$ is the QR decomposition of A, $u(A) := \operatorname{diag} U \in \mathbb{C}^n$, where $A = L\omega U$ is any Gelfand-Naimark decomposition. We obtain complete relations between (1) $u(A)$ and $a(A)$, (2) $u(A)$ and $s(A)$, (3) $u(A)$ and $\lambda(A)$, and (4) $a(A)$ and $\lambda(A)$. We also study the relations between any three elements among u, λ, a, s. This is a joint work with T.Y. Tam.

7. **Chi-Kwong Li**, ckli@math.wm.edu
Department of Mathematics, College of William and Mary, Williamsburg, Virginia, USA

Title: Numerical ranges of the powers of an operator

Abstract: The numerical range $W(A)$ of a bounded linear operator A on a Hilbert space is the collection of complex numbers of the form (Av, v) with v ranging over the unit vectors in the Hilbert space, and the numerical radius $w(A)$ of A is the radius of the smallest circle centered at the origin enclosing $W(A)$. We obtain inclusion regions for $W(A^k)$ for positive integers k and $k = -1$. From these results, some matrix and operator inequalities are derived, and the conditions for equalities are studied.

This is joint work with Man-Duen Choi (University of Toronto).

8. **Zhongshan Li**, matzli@langate.gsu.edu
Department of Mathematics & Statistics, Georgia State University, Georgia, USA

Title: Spectrally arbitrary tree sign pattern matrices

Abstract: A sign pattern (matrix) is a matrix whose entries are from the set $\{+, -, 0\}$. A sign pattern matrix A is a spectrally arbitrary pattern if for every monic real polynomial $p(x)$ of degree n there exists a real matrix B whose entries agree in sign with A such that the characteristic polynomial of B is $p(x)$. All 3×3 SAP's, as well as tree sign patterns with star graphs that are SAP's, have already been characterized. We investigate tridiagonal sign patterns of order 4. All irreducible tridiagonal SAP's are identified. Necessary and sufficient conditions for an irreducible tridiagonal pattern to be an SAP are found. Some new techniques, such as innovative applications of Gröbner bases for demonstrating that a sign pattern is not potentially nilpotent, are introduced. Some properties of sign patterns that allow every possible inertia are established.

This is joint work with M. Arav, F. Hall and K. Kaphle.

9. **Peter Nylen**, nylenpm@auburn.edu
Department of Mathematics & Statistics, Auburn University, Alabama, USA

Title: Partial distance matrices and spring network equilibrium

Abstract: Given a collection of points P_1, P_2, ... P_n in k dimensional Euclidean space, the distance matrix A generated by these points is the real symmetric matrix whose (i, j) entry is distance(P_i, P_j). Given a matrix A one may determine if it is a distance matrix, and find a set of points that generates A. A partial matrix is a matrix with some of the entries unknown. The problem of characterizing which partial matrices may be completed to distance matrices is not completely understood. We describe a scheme for finding distance matrices which are close in a minimum energy sense to being a completion for a given partial matrix. The approach is to consider the specified distances as the natural length of springs, and then compute an equilibrium position of the spring network by a Newton's Method iteration.

10. **Tom Pate**, patettho@auburn.edu
Department of Mathematics & Statistics, Auburn University, Alabama, USA

Title: A new lower bound inequality for the norm of the symmetric product

Abstract: If V is a complex finite dimensional inner product space, and q is a positive integer, then $\mathbf{T}_q(V)$ denotes the set of all symmetric q-linear $\mathbb{C}$-valued functions with domain $V\times V\times\cdots\times V$-($q$ copies), and $\mathbf{S}_q(V)$ denotes the subspace of $\mathbf{T}_q(V)$ whose elements are the fully symmetric q-linear $\mathbb{C}$-valued functions. If $A \in \mathbf{S}_n(V)$ and $B \in \mathbf{S}_p(V)$, then we define the tensor product, $A \otimes B$, and the symmetric product, $A \cdot B$, in the usual way, and we obtain an inner product, and norm, on $\mathbf{T}_q(V)$ by extending the inner product on V. If $x \in V$ and $C \in \mathbf{S}_q(V)$, then $C(x)$ denotes the member of $\mathbf{S}_{q-1}(V)$ such that $C(x)(v_1, v_2, \ldots, v_{q-1}) = C(x, v_1, v_2, \ldots, v_{q-1})$ for all $v_1, v_2, \ldots, v_{q-1} \in V$. We prove that if $\{e_i\}_{i=1}^m$ is an orthonormal basis for V, then $\|A\cdot B\|^2 \geq [n/(n+p)] \sum_{i=1}^m \|A(e_i)\cdot B\|^2$ for all $A \in \mathbf{S}_n(V)$ and all $B \in \mathbf{S}_p(V)$. This inequality improves upon Neuberger's inequality, which states that if $A \in \mathbf{S}_n(V)$ and $B \in \mathbf{S}_p(V)$, then $\|A \cdot B\|^2 \geq [1/\binom{n+p}{n}] \|A\|^2\|B\|^2$. It also implies a known refinement to Lieb's inequality, and generates several new permanental inequalities as well.

11. **Helena Smigoc**, helena.smigoc@fmf.uni-lj.si
School of Mathematical Sciences, University College Dublin, Belfield, Ireland

Title: Spectrum of nonnegative matrices

Abstract: The question which lists of complex numbers are the spectrum of some nonnegative matrix is known as the nonnegative inverse eigenvalue problem (NIEP). This problem is open for lists with more than four elements. In this talk we will give an overview of some of our recent results on this problem. In particular, we will present the solution for the case when all elements of the possible spectrum, except the dominant one, have negative real part. Best possible results are obtainable in this case and the conditions are surprisingly easy to state.

The talk is based on a joint work with Thomas Laffey.

12. **Ronald L. Smith**, ronald-smith@utc.edu
Department of Mathematics, University of Tennessee at Chattanooga, Tennessee, USA

Title: Some remarks on inverse M-matrices

Abstract: Those nonnegative matrices, some Hadamard power of which are inverse M-matrices are characterized, This requires a refinement of the strict path product necessary condition for an inverse M-matrix, first noted by R. A. Willoughby. In the process, several new results about inverse M-matrices are given.

13. **Raymond Sze**, sze@math.uconn.edu
Department of Mathematics, University of Connecticut, Connecticut, USA

Title: Optimization of the spectral radius of a product for nonnegative matrices

Abstract: Let A be an $n \times n$ irreducible nonnegative matrix. We show that over the set Ω_n of all $n \times n$ doubly stochastic matrices S, the spectral radius $\rho(SA)$ attains a minimum and a maximum at a permutation matrix. Generalization and related results will also be discussed. This talk is based on a joint work with J. Axtell, L. Han, D. Hershkowitz and M. Neumann.

14. **Wen Yan**, wy0615900@tuskegee.edu
Department of Mathematics, Tuskegee University, Alamaba, USA

Title: Unitary completions of complex symmetric and skew symmetric matrices

Abstract: Unitary symmetric completions of complex symmetric matrices are obtained via Autonne decomposition. The problem arises from atomic physics. Of independent interest unitary skew symmetric completions of skew symmetric matrices are also obtained by Hua decomposition. This is a joint work with T.Y. Tam.

15. **Fuzhen Zhang**, zhang@nova.edu
Division of Math, Science & Technology, Nova Southeastern University, Florida, USA

Title: Hua's matrix equality and Schur complements

Abstract: The purpose of this presentation is to revisit Hua's matrix equality (and inequality) through the Schur complement. We present Hua's original proof and two new proofs with some extensions of Hua's matrix equality and determinantal inequalities. The new proofs use a result concerning Schur complements and a generalization of Sylvester's law of inertia, each of which is useful in its own right. This is a joint work with C. Page, G. P. H. Styan, and B.-Y. Wang.