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ENDOGENOUS TRADE AND FACTOR PROPORTIONS PRODUCTION

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ABSTRACT

The present paper integrates neoclassical offer curves and production theory, introducing utility maximization and trade levels into the comparative static factor proportions model. The paper analyzes production conditions that determine the Metzler paradox that a tariff could lower the tariff inclusive import price. The model provides a more complete link between neoclassical and factor proportions trade models.

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INTRODUCTION

Two of the building blocks of trade theory are neoclassical and factor proportions models. The main tool of the neoclassical trade model of Marshall (1879) and Meade (1952) is the offer curve of general production and utility functions in Johnson and Bhagwati (1960), Mundell (1960), Chipman (1965), Kemp (1974), and Whitaker (1975). Dixit and Norman (1980) and Wong (1995) develop links between production and consumption with general expenditure or revenue functions. The factor proportions model developed by Samuelson (1953) and reviewed by Jones and Neary (1984) is a comparative static neoclassical production model without utility maximization or explicit trade levels. Bhagwati and Srinivasan (1983) derive offer curves for the special case of complete specialization and constant cost production but there is no explicit link between the factor proportions model and offer curves. This is unfortunate because the broad neoclassical perspective afforded by Kemp (1974) seems lost when focused on production in the factor proportions model. The present "Heckscher-Ohlin-Marshall" trade model includes utility maximization and trade in a comparative static factor proportions model, linking factor intensity, factor substitution, marginal utilities, and trade levels. The model focuses on the

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effect of changing prices on offer curves familiar from neoclassical trade theory, analyzing the production conditions that lead to the Metzler (1949) paradox that a tariff lowers the relative price of imports inclusive of the tariff.

COMPARATIVE STATICS OF PRODUCTION

The building blocks of competitive models of production and trade are constant returns to scale and full employment as developed by Jones (1965), Chipman (1966), and Takayama (1982). A fundamental condition is full employment $v_i = \Sigma_j a_{ij} x_j$ where v_i is the exogenous factor endowment, a_{ij} is a cost minimizing unit input, and x_j is the endogenous output of product j = 1,2. Shephard δc_j / δdw_i lemma $\delta c_j/\delta w_i = a_{ij}$ follows from the first order condition of minimizing cost $c = \Sigma_i a_{ij} w_i$ where w_i is the price of factor i. Fully differentiate the full employment condition to find

$$dv_i = \Sigma_k s_{ik} dw_k + \Sigma_j a_{ij} dx_j, \tag{1}$$

where s_{ik} summarizes the substitution of factor i due to a change in the price of factor k across the economy, $s_{ik} \equiv \Sigma_j x_j \, \delta a_{ij}/\delta w_k$. Young theorem on partial derivatives $\delta a_{ij}/\delta w_k = \delta a_{kj}/\delta w_i$ implies symmetric substitution terms, $s_{12} = s_{21}$. By Shepheard lemma $\delta c_j/\delta w_i = a_{ij}$ where the cost function c_j is homogeneous of degree one in factor prices w, implying the derived $a_{ij}(w)$ are homogeneous of degree zero and the related Euler equation $\Sigma_i w_i(\delta a_{kj}/\delta w_i) = 0$. Without loss of generality rescale factor prices to $w_i = 1$ and homogeneity implies $\delta a_{1j}/\delta w_1 = -\delta a_{1j}/\delta w_2$ and $s_{11} = -s_{12}$. Similarly, $s_{22} = -s_{21}$ and for simplicity $s \equiv s_{12} = s_{21} = -s_{11} = -s_{22}$. The full employment conditions are the first two equations in the comparative static system (7).

Competitive pricing implies price equals cost, $p_j = \Sigma_i a_{ij} w_i$ where p_j is the price of good j and w_i the endogenous price of factor i. Differentiate and use the cost minimizing envelope theorem to find

$$dp_j = \Sigma_i a_{ij} dw_i, \tag{2}$$

the second and third equations in (7).

Utility Maximization and Balanced Trade

The economic problem faced by consumers in a small open economy is to maximize utility subject to the terms of trade and the underlying production frontier. With production on the frontier and trade balanced, national income y derives from the tangency of the terms of trade line with the production frontier, $y = \Sigma_j p_j x_j = \Sigma_j p_j c_j$.

Constrained utility maximization with two products in Lagrangian form is

$$\Lambda = u(c_1,c_2) + \lambda(y - \Sigma_j p_j c_j). \tag{3}$$

The first order condition equates the marginal rate of substitution with the relative price, $u_1/u_2 = p_1/p_2$ where $u_j \equiv \delta u / \delta C_j$.

For tractability in the linear comparative static model, assume linear marginal utilities

$$u_m = \beta_{mn}c_n - \beta_{mm}c_m,\ m \neq n. \tag{4}$$

Positive β coefficients capture diminishing marginal productivity and symmetric complements. Linear marginal utility can be considered an approximation to the true marginal utility function. Using (4) the first order condition $u_1p_2 = u_2p_1$ becomes $\beta_{12}c_1p_1 - \beta_{22}c_2p_1 = \beta_{12}c_2p_2 - \beta_{11}c_1p_2$ which is fully differentiated to find

$$u_1dp_2 - u_2dp_1 = \gamma_1dc_1 - \gamma_2dc_2, \tag{5}$$

where $\gamma_j = \beta_2p_1 + b1_jp_2 > 0$. This first order condition of utility maximization is the fifth equation of (7).

The final condition is balanced trade, $\Sigma_jp_j(x_j - c_j) = 0$. Fully differentiating,

$$p_1dx_1 + p_2dx_2 - p_1dc_1 - p_1dc_1 = \chi_1dp_1 + \chi_2dp_2, \tag{6}$$

where excess demand is $\div_j\ \chi_i \equiv c_j - x_j$. With no loss of generality, standardize products to unit prices $p_j = 1$ to arrive at the last equation in (7).

The Comparative Static System of Production and Trade

Conditions of full employment (1), competitive pricing (2), utility maximization (5), and balanced trade (6) are combined into the comparative static system

$$\begin{pmatrix} -s & s & a_{11} & a_{12} & 0 & 0 \\ s & -s & a_{21} & a_{22} & 0 & 0 \\ a_{11} & a_{21} & 0 & 0 & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & -\gamma_2 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} dw_1 \\ dw_2 \\ dx_1 \\ dx_2 \\ dc_1 \\ dc_2 \end{pmatrix} = \begin{pmatrix} dv_1 \\ dv_2 \\ dp_1 \\ dp_2 \\ u_1dp_2 - u_2dp_1 \\ \chi_1dp_1 + \chi_2dp_2 \end{pmatrix}. \tag{7}$$

Exogenous variables are factor endowments v_i and product prices p_j, and endogenous variables are factor prices w_i, outputs x_j, and consumption levels c_j. The comparative static results of changes in endowments and prices on factor prices, outputs, and consumption are found with Cramer rule, illustrated in the Appendix. The determinant of the system matrix is $\Delta = -\gamma b^2 < 0$ where $\gamma \equiv \gamma_1 + \gamma_2$ and $b \equiv a_{11}a_{22} - a_{12}a_{21} > 0$ with factor 1 used intensively to produce good 1.

Adjustment occurs along the production frontier or contract curve. Changing endowments have no effect on factor prices, $\delta w_i/\delta v_k = 0$, the factor price equalization result from the factor proportions model.

Endowment Changes and Reciprocity

Reciprocity between the Rybczynki äx$_j$/äv$_i$ and Stolper-Samuelson äw$_i$/äp$_j$ terms holds based on factor intensity,

$$\begin{aligned} \delta x_1/\delta v_1 &= \delta w_1/\delta p_1 = a_{22}/b > 0 \\ \delta x_1/\delta v_2 &= \delta w_2/\delta p_1 = -a_{12}/b < 0 \\ \delta x_2/\delta v_1 &= \delta w_1/\delta p_2 = -a_{21}/b > 0 \\ \delta x_2/\delta v_2 &= \delta w_2/\delta p_2 = a_{11}/b > 0. \end{aligned} \tag{8}$$

Changing endowments affect consumption and trade. Focusing on changes in the endowment of factor 1, consumption adjusts according to

$$\begin{aligned} \delta c_1/\delta v_1 &= \gamma_2(a_{22} - a_{21})/\gamma b \\ \delta c_2/\delta v_1 &= \gamma_1(a_{22} - a_{21})/\gamma b. \end{aligned} \tag{9}$$

Increased endowments expand the production frontier raising national income. Consumption levels of both products increase since $a_{22} > a_{21}$ due to cost minimization with the present rescaling.

Combining adjustments in production in (8) and consumption in (9), the effects of a change in the endowment of factor 1 on the excess demand for good 1 is

$$\delta\chi_1/\delta v_1 = -(a_{22}\gamma_1 + a_{21}\gamma_2)/\gamma b < 0. \tag{10}$$

Increased excess demand for one product must be matched by increased excess supply of the other given constant prices and balanced trade, implying $\delta\chi_1/\delta v_1 = -\delta\chi_2/\delta v_1$. An increased in the endowment of the factor used intensively to produce exports raises trade, a point developed by Kemp (1964) with general functions. Following a similar proof, an increase in the endowment of the factor used intensively in import production lowers trade.

Prices and Excess Demands

Price changes affect outputs through the underlying factor proportions production structure,

$$\delta x_1/\delta p_1 = \delta x_2/\delta p_2 = -\delta x_1/\delta p_2 = -\delta x_2/\delta p_1 = s/b^2 > 0, \tag{11}$$

using the competitive pricing condition $a_{11} + a_{21} = a_{12} + a_{22} = 1$ that follows from competitive pricing with products and factors rescaled to unit prices. A higher degree of factor substitution and less factor intensity difference imply a less convex production frontier and larger $\delta x/\delta p$ effects.

Price changes also affect consumption. Consider an increase in the price of exported good 1 given $\chi_1 < 0$ and $\chi_2 > 0$. Consumption effects are

$$\begin{aligned} \delta c_1/\delta p_1 &= -(\gamma_2\chi_1 + u_2)/\gamma \\ \delta c_2/\delta p_1 &= -(\gamma_1\chi_1 - u_2)/\gamma > 0. \end{aligned} \tag{12}$$

The sign of $\delta c_1/\delta p_1$ is ambiguous. A higher export price causes substitution in consumption toward the imported product and raises income along the production frontier to the improved terms of trade line. If the marginal utility of good 2 is small $0 < u_2 < -\gamma_2\chi_1$ there is less substitution toward imported product 2 and $\delta c_1/\delta p_1 > 0$. The income effect $-\gamma_2\chi_1/\gamma$ tied to increased export production outweighs the substitution effect $-u_2/\gamma$. For the import, both the substitution and income effect work in the same direction. Similar analysis holds for the consumption effects of a change in the import price.

Combining (11) and (12) into the effects of a higher price of exported good 1 on excess demands,

$$\delta\chi_1/\delta p_1 = -[(u_2 + \gamma_2\chi_1)b^2 + \gamma s]/\gamma b^2 \quad (13)$$
$$\delta\chi_2/\delta p_1 = [-(u_2 - \gamma_1\chi_1)\, b^2 + \gamma s]/\gamma b^2 > 0.$$

A higher export price unambiguously raises imports as decreased production complements increased consumption. Ambiguity arises, however, in the net effect on exports due to the income effect.

Exports fall with an increase in the excess demand for good 1, the negative χ_1 increasing and $\delta\chi_1/\delta p_1 > 0$. From (13), four influences favor reduced exports and a backward bending offer curve:

(i) a higher export level
(ii) lower marginal utility of imports
(iii) larger factor intensity difference
(iv) less substitution in production.

At higher export levels, an increased export price generates a larger increase in income and income effect. Lower marginal utility of the import favors consumption of the export. A larger difference in factor intensity and less substitution in production both imply a more concave production frontier and a less of an increase in export production with its higher price. The effect of a change in the price of good 2 on the excess demand for good 1 is symmetric to (13),

$$\delta\chi_1/\delta p_2 = [(u_1\ \gamma_2\chi_2)b^2 + (\gamma + \gamma_2)s]/\gamma b^2. \quad (14)$$

Since $\chi_2 > 0$, $\delta\chi_1/\delta p_2$ could be negative. Influences favoring a positive $\delta\chi_1/\delta p_2$ include a higher import level and lower marginal utility of the export. Offer curves are analyzed in terms of relative prices, combining the effects of changes in p_1 and p_2. A backward bending offer curve is favored by a higher trade level, lower marginal utilities, larger factor intensity difference, and less substitution in production.

TARIFFS AND TRADE

Consider the effects of a home tariff facing the foreign country in (13) and (14). The Metzler paradox occurs if the foreign offer curve bends backwards, the tariff causing not only improved terms of trade but also increased imports and decreased exports.

Kemp (1964, pp. 33-4) and Minabe (1974) describe the neoclassical conditions for a backward bending offer curve. Metzler (1949) finds that a tariff lowers the domestic relative price including the tariff if $\varepsilon < \mu/(\mu - 1)$ where μ is the marginal propensity to import and ε is the foreign offer curve elasticity. Specifically, ε is the percentage change in home exports due to a 1% increase in home imports, and if $\varepsilon < 0$ the offer curve has negative slope. Since $0 < \mu < 1$, it follows that $\mu/(\mu - 1) < 0$ and the Metzler condition follows.

In the present model, the level of trade, marginal utilities, factor intensity, and substitution in production influence the slope of the offer curve and determine whether the Metzler condition holds. Local comparative static conditions in (13) and (14) isolate the microeconomic conditions of production and consumption underlying the paradox. The present model, in other words, provides the microeconomic foundation of the offer curve elasticity å and marginal propensity to import μ.

CONCLUSION

The present comparative static factor proportions model of offer curves links neoclassical trade theory directly to factor proportions production. It is possible to extend the present parametric model to include specific factors of production, internationally mobile factors, various industrial structures, and other relaxations suggested by Thompson (2003). Regarding a dramatic application, OECD oil consuming countries might gain from a tariff that would lower the tariff inclusive price, raise imports, and reduce exports to pay the fuel bill. Oil exporting OPEC certainly meets the conditions of a backward bending offer curve with its high trade level, low marginal utilities due to satiation, and oil production that is highly capital intensive with no input substitution.

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MATHEMATICAL APPENDIX

As an example of the comparative static derivation, consider the solution for $\delta c_1/\delta p_1$ using Cramer's rule. First let $dp_2 = dv_1 = dv_2 = 0$ in (7) and divide through by dp_1. The resulting exogenous vector is then the fifth column in the cofactor matrix below. To follow the derivation, expand up the last row

$$(\delta c_1/\delta p_1)\Delta = \begin{vmatrix} -s & s & a_{11} & a_{12} & 0 & 0 \\ s & -s & a_{21} & a_{22} & 0 & 0 \\ a_{11} & a_{21} & 0 & 0 & 1 & 0 \\ a_{12} & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_2 & -\gamma_2 \\ 0 & 0 & 1 & 1 & \chi_1 & -1 \end{vmatrix}$$

$$= u_2 b^2 + \gamma_2 \chi_1 b^2 + \gamma_2(a_{11} + a_{12})[(a_{11} + a_{21}) - (a_{12} + a_{22})]$$

$$= (\gamma_2 \chi_1 + u_2) b^2$$

with $a_{11} + a_{21} = a_{12} + a_{22} = 1$ due to competitive pricing and rescaled prices. It follows that $\delta c_1/\delta p_1 = (u_2 + \gamma_2\chi_1)/{-\gamma}$ as in (12).