Economic Growth with a Nonrenewable Resource

Henry Thompson

Auburn University

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Abstract. This paper adds a nonrenewable resource to capital and labor in the neoclassical growth model. The nonrenewable resource introduces its depletion dynamics and expands the influence of input substitution on the growth path. Optimal depletion implies a rising resource price but investment or labor growth may raise extraction along the growth curve. Substitution between inputs plays a critical role in the model dynamics. The paper develops the fundamental conditions for intergenerational equity, and also examines the tragedy of the commons and a myopic resource owner.

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A nonrenewable resource added to the neoclassical growth model with capital and labor introduces its own dynamics and expands the role of substitution in economic growth and income distribution. This paper examines growth along the phase curve with the depleting resource and transitory adjustments in the resource price, capital return, wage, income per capita, and input ratios. Optimal depletion implies a rising resource price but investment and labor growth increase resource productivity and may raise depletion. Resource demand would increase if the capital return is falling and the resource is a complement with capital. Depletion would also increase with a rising wage if the resource is a substitute for labor. The paper also examines conditions for intergenerational equity.

The paper derives the transitional dynamics with optimal depletion and substitution in a general production function for increasingly complex models. The paper also examines the tragedy of the commons and a myopic owner of the nonrenewable resource.

1. Transitional dynamics in the neoclassical growth model

Capital and labor are the only inputs in the constant returns production function $Y = Y(K, L)$ of the neoclassical growth model of Solow (1956) and Swan (1956). The per capita production function is $y = f(k)$ where $y \equiv Y/L$ and $k \equiv K/L$. Investment changes the capital stock according to $dK/dt = K'$. Saving S equals investment assuming a closed economy with no depreciation. Constant marginal propensity to save σ implies $K' = \sigma Y$ assumed to be the golden rate in the present paper. The labor force grows at the constant rate $\lambda \equiv L'/L$.

The capital/labor ratio changes according to $(K/L)' = \sigma Y/L - \lambda k$ or

$$k' = \sigma y - \lambda k. \tag{1}$$

Divide national income $Y = wL + rK$ by L to find $y = w + rk$. Output changes through marginal products according to $Y' = Y_L L' + Y_K K' = wL' + rK'$ with factors paid marginal products in competitive markets. Divide through by L to find $Y'/L = y' + \lambda y = \lambda w + r(k' + \lambda k)$ leading to

$$y' = rk'. \tag{2}$$

Intergenerational equity or constant income per capita occurs only in the steady state where $y' = 0 = k'$.

There are numerous extensions and refinements in the literature. To mention only a few, Solow (1974) and Stiglitz (1974) show that per capita income falls in the steady state with exponential labor growth. Dasgupta and Heal (1979) examine optimal growth paths. Robinson (1980) shows how properties of the model relate to classical economics.

2. The dynamics of depleting a nonrenewable resource

Extraction N of the nonrenewable resource diminishes its stock S according to $N = -S'$. One simple assumption is that a fixed share α of the stock S is depleted each year, $N = \alpha S$ implying changes in depletion are directly related to the level of depletion, $N' = \alpha S' = -\alpha N$ or $N'/N = -\alpha$.

Assuming constant returns, per capita income is a function of k and the resource/labor ratio $h \equiv N/L$ in the per capita production function $y = f(k, h)$. Capital deepening with a rising k can offset the effect of resource exhaustion on per capita income as h falls. With a constant depletion rate, the convergent resource/labor ratio evolves according to $h' = (N/L)' = N'/L - \lambda h = -(\alpha + \lambda)h < 0$. As $h \to 0$ income per capita approaches the KL steady state $y^s = \lambda k/\sigma$.

Changes in national income $Y = wL + rK + nN$ occur according to $Y' = wL' + rK' + nN'$ assuming factors are paid marginal products. Divide by L to find $y = w + rk + nh$ and $Y'/L = y + \lambda y = \lambda w + r(k' + \lambda k) + n(h' + \lambda h)$ leading to

$$y' = rk' + nh'. \tag{3}$$

The presumption is for a positive k' with falling r and rising w as in the K/L model. The rising resource price, however, increases capital demand if the two are substitutes and offsets the increasing capital. Demand for labor would fall if it were a complement with the resource. Substitution then determines the growth path and associated income distribution.

Leaving aside the constant depletion rate, the present paper assumes optimal depletion with a perfect asset market between capital and the resource. Ownership of either asset represents future income. Resource owners with known reserves and perfect foresight deplete to satisfy the Hotelling (1931) condition equalizing returns to the two assets.

With zero marginal extraction cost MEC, optimal depletion implies the no-arbitrage condition

$$n'/n = r. \qquad (4)$$

Implications and extensions of this optimal depletion condition are developed by Dixit, Hammond, and Hoel (1980), Hamilton (1995), Withagen and Asheim (1998), and Sato and Kim (2002).

3. Substitution in the three factor KLN model

Substitution between the three inputs is based on Allen (1938) as developed by Takayama (1982, 1993), Jones and Easton (1983), and Thompson (1985, 2006). Endogenous depletion equals resource demand, $N = a_N Y$. In per capita terms, $h = a_N y$. The cost minimizing resource input per unit of output is a function of factor prices, $a_N(r, w, n)$. With homothetic production a_N varies only with input prices.

Adjustment in resource input is written $h' = y a_N' + a_N y'$. Expand a_N' across changes in factor prices and introduce input substitution to find

$$h' = N_r r' + N_w w' + N_n n' + a_N y'. \qquad (5)$$

Substitution terms describe how inputs adjust to input prices. For the resource, $N_r \equiv (\partial a_N/\partial r)y$ represents cross price substitution with respect to the price of capital, $N_w \equiv (\partial a_N/\partial w)y$ substitution with respect to the wage, and $N_n \equiv (\partial a_N/\partial n)y$ with respect to the own price. Capital substitution terms K_r, K_w, and K_n and labor substitution terms L_r, L_w, and L_n are similar. Together, these substitution terms describe the local surface of the production isoquant. With three inputs, the production surface may be concave in one direction.

Cost minimization and Shephard's lemma imply unit inputs a_i are partial derivatives of the unit cost function with respect to each price in the expression $a_N = \partial c/\partial n$. Substitution terms are symmetric by Young's theorem. For instance, $N_r = \partial^2 c/\partial n \partial r = K_n$. The unit cost function c(r, w, n) is homogeneous of degree one in factor prices implying unit inputs are homogeneous of degree zero. Euler's theorem implies $nN_n + rN_r + wN_w = 0$ with the own term N_n derived from the two cross terms. Given symmetry and Euler's theorem, the three independent substitution terms are N_r, N_w, and K_w.

The second derivative of the cost function is negative with respect to each input, implying the negative own substitution terms N_n, K_r, and L_w. Substitution terms are positive between substitutes but one pair of inputs may be complements with a negative substitution term.

The related substitution elasticities can be derived, for instance $\varepsilon_{Nr} \equiv (r/N)N_r$. With Cobb-Douglas substitution, there are unit Allen elasticities $\sigma = 1$ implying the cross price elasticities are equal to factor shares in the condition $\varepsilon_{ij} = \sigma\theta_j = \theta_j$. Other production functions have potential for stronger or weaker substitution, as well as one pair of input complements.

4. Transitional dynamics in the two factor KL and KN models

This section presents the relatively simple transitional dynamics for the capital/labor and capital/resource models before moving to the three input model. The first equation in the neoclassical KL model (6) is capital employment, and the second labor employment. The third equation is changes in income per capita from (3) and the last capital deepening,

$$\begin{pmatrix} K_r & K_w & a_K & -1 \\ L_r & L_w & a_L & 0 \\ 0 & 0 & 1 & -r \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r' \\ w' \\ y' \\ k' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sigma y - \lambda k \end{pmatrix} . \tag{6}$$

The determinant $\Delta_{KL} = K_r L_w - K_w L_r$ is positive given the regularity condition that own effects outweigh cross effects. Solutions to (6) are

$$\begin{aligned} r' &= -k'(-r(a_L K_w - a_K L_w) - L_w)/\Delta_{KL} < 0 \\ w' &= k'(r(a_K L_r - a_L K_r) - L_r)/\Delta_{KL} > 0 \\ y' &= rk'. \end{aligned} \tag{7}$$

Intergenerational equity $y' = 0$ occurs only in the steady state where $k' = 0$.

The KE model with capital and resource inputs in (8) has employment conditions for resource in the first equation, and for capital in the second equation. The third equation reflects changes in income $Y' = rK' + nN' + Kr' + Nn'$ assuming cost minimization $rK' + nN' = 0$ and optimal depletion $n' = rn$,

$$\begin{pmatrix} N_r & a_N & -1 \\ K_r & a_K & 0 \\ -K & 1 & 0 \end{pmatrix} \begin{pmatrix} r' \\ Y' \\ N' \end{pmatrix} = \begin{pmatrix} -rnN_n \\ -rnN_r + \sigma Y \\ rnN \end{pmatrix} . \tag{8}$$

The negative determinant is $\Delta_{NK} = -(K_r + a_K K)$. Directions of change in the capital return and income are

$$r' = (rn(a_K N + N_r) - \sigma Y)/\Delta_{NK} \tag{9}$$

$$Y' = [rn(KN_r - NK_r) - \sigma YK]/\Delta_{NK}$$

Assuming $r' < 0$ higher depletion N implies less of a decrease in r due to the increased capital productivity. Higher substitution also implies less of a decline in r, while a higher saving rate favors more of a decrease in r.

Regarding the adjustment in income, the expression $KN_r - NK_r$ is positive. Increased income is favored by higher saving and a higher capital stock. Higher substitution favors less of an increase in income. The expression for extraction N′ is difficult to analyze and not reported.

With Cobb-Douglas production, the cross price substitution terms are $K_n = KN/Y$ and $K_r = K(rK - Y)/rY$. Substituting into (9) intergeneration equity $Y' = 0$ occurs iff $\sigma Y = nN$. This result is the Hartwick (1977) rule to invest all resource revenue for intergenerational equity. In a related two factor model, Garg and Sweeney (1978) derive optimal growth paths dependent on initial endowments.

5. Transitional dynamics in the three factor KLN model

The first equation of the KLN model (10) is resource employment in the resource/labor ratio h′. The second equation is a similar condition for capital where $k' = \sigma y - \lambda k$. The third equation is labor employment. Per capita income adjusts in the fourth equation according to (3). Capital accumulation is the last equation in the system

$$\begin{pmatrix} N_r & N_w & a_N & -1 & 0 \\ K_r & K_w & a_K & 0 & -1 \\ K_w & L_w & a_L & 0 & 0 \\ 0 & 0 & 1 & -n & -r \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r' \\ w' \\ y' \\ h' \\ k' \end{pmatrix} = \begin{pmatrix} -rnN_n \\ -rnN_r \\ -rnN_w \\ 0 \\ \sigma y - \lambda k \end{pmatrix}. \qquad (10)$$

For simplicity n′ and c′ are omitted in the system (10) but can be derived. The increase in the resource price is derived from the optimal depletion condition according to $n' = rn$. Consumption is tied directly to income in the condition $c' = (1 - \sigma)y'$.

The comparative static system solves for endogenous instantaneous changes in the capital return r′, wage w′, per capita income y′, resource/labor ratio h′, and capital/labor ratio k′. These directions of change depend on substitution as well as the levels of inputs, output, and factor prices. Regularity conditions in production restrict substitution according to

$$d_1 \equiv N_nK_r - N_r^2 > 0 \qquad d_2 \equiv N_nL_w - N_w^2 > 0 \qquad d_3 \equiv K_rL_w - K_w^2 > 0 \tag{11}$$

$$d_4 \equiv N_wN_r - N_nK_w > 0 \quad d_5 \equiv N_rK_w - K_rN_w > 0 \quad d_6 \equiv N_wK_w - N_rL_w > 0.$$

The sign of the determinant $\Delta = -n(a_Nd_3 + a_Ld_5 + a_Kd_6) + d_3$ depends on substitution as well as unit input levels and the resource price n. Instability of the system with a zero determinant is a global issue. The present paper assumes local stability.

Including the condition for capital deepening, cofactors of solutions for endogenous variables are

$$r' = r[-n^2(a_Kd_2 + a_Ld_4 + a_Nd_6) - nd_6 + k'(a_LK_w - a_KL_w)]$$

$$w' = r[-n^2(a_Ld_1 + a_Kd_4 + a_Nd_5) + nd_5 - k'(a_LK_r - a_KK_w)] \tag{12}$$

$$y' = r[n^2(N_nd_3 + N_wd_5 + N_rd_6) + k'd_3]$$

$$h' = r[n(N_nd_3 + N_wd_5 + N_rd_6) + k'(a_Nd_3 + a_Ld_5 + a_Kd_6)].$$

Each may be positive or negative, but there are presumptions that $r' < 0$, $w' > 0$, $y' > 0$, and $h' < 0$. Assuming a negative determinant, a higher saving rate implies a larger k′ with a smaller r′ and larger w′ and y′.

The following simulations do not exhaust possibilities but provide some insight into model dynamics. With no loss of generality, scale inputs and output to one implying unit inputs $a_i = 1$.

Assume $\sigma = 0.12$ and $\lambda = 0.02$ implying $k' = 0.1$. Focus on resource substitution assuming $K_w = 1$ with Cobb-Douglas substitution between capital and labor. Assume the factor price vector (w, r, n) = (0.4, 0.4, 0.2) that equals factor shares with the scaling. Suppose $N_r = 1$ and let N_w vary from substitutes to complements. If $N_w = 0.5$ the vector of endogenous adjustments is (r′, w′, y′, h′) = (0.86, 0.84, 0.36, 1.32). The rising resource price increases labor demand, raising the wage. Demand for capital increases as does the capital return in spite of capital deepening. Income per capita rises. The resource/labor ratio increases as extraction outpaces labor growth. If the resource and labor are complements with the substitution term $N_w = -0.5$, the adjustment vector is (0.22, 0.17, 0.08, -0.11). The rent and wage increase but much less than when the resource and labor are substitutes. There is very little increase in per capita income. The resource/labor ratio falls consistent with the smaller increase in the rent and wage.

Now consider substitution between the resource and capital in the N_r term. Assume $K_w = 1$ and $N_w = 0.1$. If $N_r = 1$ the vector of adjustments is (0.42, 0.40, 0.16, 0.30). If N_r equals -0.1 the vector is (0.50, 0.40, 0.09, -0.07). With less substitution for capital, the rising resource price implies more of an increase in capital rent and a diminishing resource/labor ratio.

Intergenerational equity with $y' = 0$ is satisfied only if $n^2(N_n d_3 + N_w d_5 + N_r d_6) = -k' d_3$, a completely accidental condition. Investing resource rent is not relevant to intergenerational equity. At any rate, developing economies would target rising per capita income. In practice, knowledge of production is essential with no simple rule to follow for intergenerational equity.

6. The tragedy of the commons and a myopic nonrenewable resource owner

Common pool ownership leads to the tragedy of the commons with over-depletion of the nonrenewable resource. There is no profit from extraction as owners of the common pool drive price to marginal extraction cost MEC. For simplicity, assume MEC = 0, implying the clear tragedy where $n = 0 = n'$. The comparative static model with common pool ownership is then similar to (10) with n equal to 0 and the exogenous vector all zeroes except for the last term $\sigma y - \lambda k$. The wage and capital return could rise or fall. Depletion per capita grows with the capital/labor ratio according to $h' = rk'$. Depletion $N' = (h' + \lambda h)L$ increases if $h' > -\lambda h$.

A myopic monopoly resource owner maximizes immediate profit disregarding the asset value of the stock, setting marginal revenue MR equal to marginal extraction cost MEC. Total resource revenue is nN implying $MR = \partial(nN)/\partial N = n + N(\partial n/\partial N) = n + N(n'/N')$. If MEC = 0 it follows that $n'N = -nN'$ with depletion offsetting the rising resource price. Constant MEC would imply constant marginal profit. The change in the resource/labor ratio is $h' = N'/L - \lambda h$. Myopic extraction $N' = -n'N/n$ implies $h'/h = -(n'/n + \lambda)$. Assuming no labor growth, the resource input offsets price implying a constant resource share of income. Labor growth would imply a falling resource share, the result of disregarding future resource revenue.

8. Conclusion

A nonrenewable resource added to capital and labor in the neoclassical growth model introduces its own dynamics and increases the potential effects of substitution on growth and income distribution. Depletion may intermittently increase along the growth path, even when the resource price is rising. The wage may fall and the capital return may rise, even with capital deepening. Adjustments along the growth path are determined by the pattern of substitution as well as the levels and prices of the three inputs. Unlike the two factor model, investing resource rent is irrelevant to maintaining per capita income.

The present three factor growth model provides a foundation for economic growth and macroeconomics for countries rich in nonrenewable resources. Simulations of the present model would reveal the evolution of depletion and income distribution for particular production functions, initial endowments, saving propensities, and labor growth rates. Model simulations can also examine global stability. Simulated growth paths can reveal turning points in depletion and income distribution as well as switch points to backstop renewable resources. Another extension of the model would be to open the economy to an exogenous international resource price.

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