Bilateral Factor Abundance and Intensity with Many Factors, Products, and Countries

Henry Thompson

Dajun Tuo

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Whether countries export products that use their abundant factors intensively depends on how these terms are defined when there are more than two factors, products, and countries since factor abundance and intensity are ratios. The present note proposes bilateral measures of factor abundance and intensity for high dimensional data based directly on the two dimensional definitions.

Do countries export products that use their abundant factors intensively? The scientific status of this fundamental proposition from factor proportions trade theory hinges on data that include various factors of production, a tremendous number of products, and many countries.

The classic factor content studies of Leontief (1954) and Baldwin (1971) include no measure of factor abundance as emphasized by Bowen, Leamer, and Sveikauskas (1987), Leamer (1994), and Thompson (1999). High dimensional factor proportions trade theory derives various generalities but falls short of defining factor abundance or intensity as in Jones (1961), Jones and Scheinkman (1976), Chipman (1979), Chang (1979), Ethier (1984), Thompson (1985, 2004), Jones (2004), and Choi (2004). The empirical factor content literature is focused on the single labor input as pointed out by Stern (1975), Deardorff (1984), and Leamer and Levinsohn (1995). There are, however, at least eight labor skill groups in US manufacturing as shown by Clark, Hofler, and Thompson (1988) and aggregation bias illustrated by Thompson (2005) raises doubts about empirical results in the factor content literature.

There are four measures of high dimensional factor abundance in the factor content literature. *Share abundance* of Vanek (1968) assumes factor price equalization. *World abundance* of Leamer (1980) is the portion of the world endowment. Rassekh and Thompson (2002) show *mean weighted factor abundance* better explains trade than either share or world abundance in the Trefler (1985) data set. The *Euclidean distance abundance* of Thompson (2003) defines the abundance of a factor relative to every other factor and performs better than the other three measures as shown by Kang, Malki, Rassekh, and Thompson (2005).

The present note proposes an alternative measure of factor abundance and factor intensity based directly on the two dimensional bilateral concept. Elements of the derived high dimensional abundance and intensity matrices are bilateral comparisons for every pair of factors with every pair of products (intensity) and countries (abundance). These measures of *bilateral factor intensity* and *bilateral factor abundance* provide a framework to analyze the empirical proposition that countries export products using their abundant factors intensively for data with any number of factors, products, and countries.

1. Bilateral factor intensity and factor abundance for high dimensional data

Introducing notation,

r	number of productive factors indexed by i, g
n	number of products indexed by j, h
c	number of countries indexed by k, m
q_{jk}	revenue from production of product j in country k
v_{ijk}	total input of factor i for product j in country k
$v_{ik} \equiv \Sigma_j v_{ijk}$	endowment of factor i in country k
$V \equiv (v_{ik})$	rxc world endowment matrix
$A_k \equiv (v_{ijk}/q_{jk}) \equiv (a_{ijk})$	rxn unit input matrix in country k
c_{jk}	consumption spending on product j in country k
$x_{jk} \equiv q_{jk} - c_{jk}$	net export revenue of product j in country k.

Factor i is intensive in product j relative to factor g and product h if $a_{ij}/a_{gj} > a_{ih}/a_{gh}$ and the positive determinant

$$a_{ig}^{jh} \equiv a_{ij}a_{gh} - a_{ih}a_{gj} \qquad (1)$$

of the 2x2 matrix transforms bilateral factor intensity into a scalar. Collect each of these a_{ig}^{jh} into a bilateral intensity matrix A_M. The elements of A_M are bilateral factor intensities.

The input matrix A has dimension rxn and the dimension of A_M is [r(r - 1)/2] x [n(n - 1)/2] with pairs of factors in each row and pairs of products in each column. The location of element a_{ig}^{jh} in A_M is (row, column) = (i(r - i - 1)/2 - (r - g), j(n - j - 1)/2 - (n - h)) where i < g and j < h. As an example the intensity matrix with three factors and three products is

$$A_M^{3x3} = \begin{pmatrix} a_{12}^{12} & a_{12}^{13} & a_{12}^{23} \\ a_{13}^{12} & a_{13}^{13} & a_{13}^{23} \\ a_{23}^{12} & a_{23}^{13} & a_{23}^{23} \end{pmatrix}.$$

A similar bilateral abundance matrix V_M is constructed from the rxc world endowment matrix V composed of 2x2 sub-determinants. Its dimensions are [r(r - 1)/2] x [c(c - 1)/2]. Each component v_{ig}^{km} of V_M reflects the abundance between factors i and g and countries k and m,

$$v_{ig}^{km} \equiv v_{ik}v_{gm} - v_{im}v_{gk}. \quad (2)$$

The location of element v_{ig}^{km} in the bilateral abundance matrix V_M is (i(r - i - 1)/2 - (r - g), k(c - k - 1)/2 - (c - m)) where i < g and k < m.

The bilateral intensity matrix A_M includes a comparison of every pair of factors across every pair of products, and the abundance matrix V_M includes a comparison of every pair of factors across every pair of countries. The following section introduces the matrix of net exports and proposes a test of the proposition that countries export products using their abundant factors intensively.

2. Testing the bilateral factor content proposition in high dimensional data

Suppose factor i is abundant in country k relative to factor g and country m, in the present notation $v_{ig}^{km} > 0$. Suppose also factor i is intensive in product j relative to product h, that is $a_{ig}^{jh} > 0$. The narrow factor content issue for these factors and products is whether country k takes advantage of its bilateral abundance in factor i relative to factor g with net exports to country m of product j relative to product h. If country k produces a higher ratio of product j to product h than country m, the net export revenue ranking is $x_{jk} > 0 > x_{hk}$ given the underlying assumptions of equalized prices by trade and homothetic demand as developed by Ruffin (1977). This narrow factor content result is not necessary in the present high dimensional model for arbitrary factors, products, and countries as Thompson (2001) shows with for three factors, products, and countries. Nevertheless, factor content might be expected to hold as a correlation.

The two dimensional net export revenue term

$$x_{jh}^{km} \equiv (x_{jk} - x_{hk}) - (x_{jm} - x_{hm}) \quad (3)$$

summarizes net trade in products j and h between countries k and m. In the 2x2x2 factor proportions model $v_{12}^{12} > 0$ implies $x_{12}^{12} > 0$. In the present high dimensional context a positive v_{ig}^{km} does not imply a corresponding positive x_{jh}^{km} but positive correlation might be expected across the data.

Collect net export terms x_{jh}^{km} into the [n(n - 1)/2] x [c(c - 1)/2] net export matrix

$$X \equiv (x_{jh}^{km}) \tag{4}$$

with pairs of products in rows and pairs of countries in columns. The x_{jk} terms can be scaled relative to GDP to eliminate the issue of different units of measure and make x_{jh}^{km} an index.

Multiply the bilateral intensity measure a_{ig}^{jh} in (1) by the bilateral abundance measure v_{ig}^{km} in (2) to derive the bilateral factor proportions scalar

$$z_{ig}^{jhkm} \equiv a_{ig}^{jh} v_{ig}^{km} \tag{5}$$

that summarizes the relationship between factors i and g for products j and h between countries k and m. A larger factor proportions term z_{ig}^{jhkm} indicates either higher intensity of factor i relative to factor g between products j and h or higher abundance between countries k and m. Either higher intensity or abundance would encourage exports.

The relationship between products j, h and countries k, m is the sum of these z_{ig}^{jhkm} terms across factor pairs,

$$z_{jh}^{km} \equiv \Sigma_{ig} z_{ig}^{jhkm} \tag{6}$$

where Σ_{ig} refers to the sum across unique product pairs. In the 3x3 model the three terms are ig = 12, 13, 23 and with r factors there are r(r - 1)/2 unique factor pairs.

The bilateral factor proportions matrix

$$Z \equiv (z_{jh}^{km}) \tag{7}$$

has elements from unique factor pairs in each column. In matrix notation,

$$Z = A_M' V_M. \tag{8}$$

With n products and c countries, the bilateral intensity abundance matrix Z has dimension [n(n - 1)/2] x [c(c - 1)/2] as does the net export matrix X in (4).

A Mantel (1967) matrix correlation or nonparametric sign test between Z and X would provide tests of the proposition that countries tend to export products using their abundant factors intensively.

3. Conclusion

The present bilateral measures of factor abundance and intensity for high dimensional data can directly test the proposition that countries export products using their abundant factors intensively. While empirical studies of factor proportions trade theory have focused on two factors, two products, or two countries, theory provides no prediction of production or exports with as few as three factors, products, and countries. The factor content proposition is then an empirical issue and the present note provides a measure to test it.

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