THE MAGNIFICATION EFFECT WITH THREE FACTORS

Henry THOMPSON

Abstract: Price changes create a range of factor price adjustments in the three factor, two good general equilibrium model of production. Comparative static adjustments in the model have been described by magnification effects and sign patterns. This note supplements the diagrammatic technique of Jones and Easton (1983, *Journal of International Economics*) and derives eleven magnification effects implied by the sign patterns of Thompson (1985, *Canadian Journal of Economics*). At the bottom line, only one type of magnification effect can be ruled out. Stated in terms of the extreme or most intensive factors, the Stolper-Samuelson theorem cannot be reversed.

1. THE MAGNIFICATION EFFECT WITH THREE FACTORS

There is ample motivation for wanting to develop general understanding of the structure of production when there are three factors of production. The three factor model with capital, labor, and land is the basis of the "classical" model of trade. Including the third factor also allows the input of energy or skilled labor. The third factor, however, complicates analysis. The important concept of factor intensity has to be reinterpreted. The possibility of technical complementarity arises. Also, factor substitution plays a more critical role in comparative statics than when there are only two primary productive factors.

Jones and Easton (1993) utilize a diagrammatic approach to derive magnification effects between percentage changes in prices of goods and factor prices in the three factor, two good (3×2) general equilibrium model of production. Thompson (1985) derives the comparative static sign patterns of the same model. This note derives all possible magnification effects implied by the comparative static sign patterns.

The third factor creates a middle term in the factor intensity ranking across industries. Ruffin (1981) shows that the most intensive or extreme factors in the ranking are migration enemies in that an increase in one factor's endowment lowers the other's price. Additionally, the middle factor is a friend of both extreme factors. Batra and Casas (1976) argue that the price of an extreme factor is positively related with the price of its good, and negatively related with the price

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of the other good. Suzuki (1983) shows that this strong Stolper-Samuelson result is not necessary. Takayama (1982) generalizes the 3×2 model, developing special cases and applications.

In the 2×2 model, percentage changes in factor prices flank percentage price changes in the Stolper-Samuelson order, according to the Jones magnification effect. In the small step to the 3×2 model, there are eleven magnification effects! At the bottom line, percentage changes in the prices of extreme factors cannot flank percentage changes in the prices of goods in the opposite order. The Stolper-Samuelson theorem, stated in terms of extreme factors, cannot be reversed.

2. SIGN PATTERNS

For consistency and convenience, completely adopt the notation of Jones and Easton (1983). The three factors are ranked by their intensity between the two sectors,

$$a_{11}/a_{12} > a_{31}/a_{32} > a_{21}/a_{22}\,. \tag{1}$$

Factor 1 is extreme in sector 1, factor 2 is extreme in sector 2, and factor 3 is the middle factor. Let p_j $(j=1,2)$ represent the exogenous price of good j and w_i $(i=1,2,3)$ the endogenous price of factor i. Where $\hat{}$ represents percentage change, the $\omega_{ij} \equiv \hat{w}_i/\hat{p}_j$ results are of interest:

$$\omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \\ \omega_{31} & \omega_{32} \end{bmatrix}. \tag{2}$$

General properties of ω follow from convexity and are developed by Jones and Scheinkman (1977). Each column of ω has at least one positive and one negative element. Each row of ω has at least one positive element. Proofs of another theorem appear in both Jones (1985) and Thompson (1985),

> *If a factor price is negatively related with the price of its extreme good, both other factor prices cannot be positively related with that price.*

These properties lead directly to the seven possible sign patterns (SPs) of ω, derived by Thompson (1985):

$$\underset{\text{a}}{\begin{bmatrix} + & - \\ - & + \\ + & + \end{bmatrix}} \underset{\text{b}}{\begin{bmatrix} + & - \\ - & + \\ + & - \end{bmatrix}} \underset{\text{b}'}{\begin{bmatrix} + & - \\ - & + \\ - & + \end{bmatrix}} \underset{\text{c}}{\begin{bmatrix} + & + \\ - & + \\ + & - \end{bmatrix}} \underset{\text{c}'}{\begin{bmatrix} + & - \\ + & + \\ - & + \end{bmatrix}} \underset{\text{d}}{\begin{bmatrix} - & + \\ - & + \\ + & - \end{bmatrix}} \underset{\text{d}'}{\begin{bmatrix} + & - \\ + & - \\ - & + \end{bmatrix}}. \tag{3}$$

All but these seven of the $2^6=64$ potential SPs of ω are eliminated by the technical properties. Structurally similar SPs occur when arbitrary names of extreme factors and goods are switched in b and b′, c and c′, and d and d′.

The specific factors model is represented by a. A strong Stolper-Samuelson effect occurs in the top two rows of a, b, and b′. An extreme factor price is positively related with both prices in c and c′. In d and d′, an extreme factor price is negatively related with the price of "its" good, which is perhaps the most surprising possibility in (3).

3. DIAGRAMMING CRAMER'S RULE

Jones and Easton picture the effects of changing prices on factor prices in two dimensions by holding the price of middle factor 3 constant ($\hat{w}_3=0$). Consider their equations (19) which express conditions of competitive pricing and full employment,

$$\begin{bmatrix} \theta_{11} & \theta_{21} & \theta_{31} \\ \theta_{12} & \theta_{22} & \theta_{32} \\ \xi_1 & \xi_2 & \xi_3 \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{w}_3 \end{bmatrix} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{v}_3-(\alpha_1\hat{v}_1+\alpha_2\hat{v}_2) \end{bmatrix}. \tag{4}$$

Each factor share term θ_{ij} represents the payment share of factor i in sector j. The endowment of factor i is represented by v_i. Relative fractions of factors used in either sector are α_1 and α_2. Substitution across the economy between the price of factor i and the input of factor k is summarized by the substitution term

$$\sigma_k^i \equiv \textstyle\sum_j x_j\,\partial a_{kj}/\partial w_i\,. \tag{5}$$

The ξ_i are *mutadis mutandis* cross price elasticities of demand for middle factor 3 with respect to the price of factor i across the economy,

$$\xi_i \equiv \sigma_3^i-(\alpha_1\sigma_1^i+\alpha_2\sigma_2^i)\,. \tag{6}$$

Schedules used in the diagrammatic approach are pictured in Fig. 1. The p_j ($j=1,2$) schedule sets $\hat{p}_j=0$. With $\hat{w}_3=0$, it follows from (4) that $\hat{w}_2/\hat{w}_1=-\theta_{1j}/\theta_{2j}<0$. Both p_j schedules thus slope downward. Note that the p_1 schedule is steeper than the p_2 schedule due the factor intensity in (1).

Full empolyment occurs along another schedule not pictured in Fig. 1. The elasticity of the full employment schedule from the last equation in (4) is equal to $-\xi_1/\xi_2$. Depending on the pattern of factor substitution, the full empolyment schedule may have positive or negative slope.

The two schedules p_1 and p_2, along with the full employment schedule, determine equilibrium prices of factors 1 and 2 at their common intersection. The full employment schedule goes through the intersection of p_1 and p_2 in the equilibrium in Fig. 1. The ray w indicates the static equilibrium ratio w_2/w_1.

Suppose there is an increase in p_1 relative to p_2. Exogenous price changes are chosen so the p_1 and p_2 schedules intersect along the implicit full employment schedule. The new static equilibrium may occur in any of the seven regions of Fig. 1.

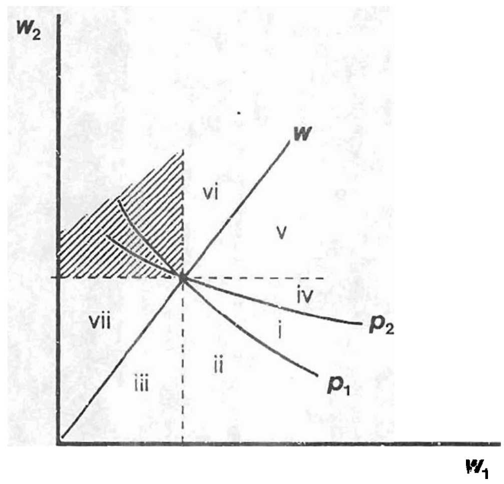

Fig. 1

Since price changes are weighted averages of factor price changes, there have to be factors h and k such that

$$\hat{w}_h > \hat{p}_1 > \hat{p}_2 > \hat{w}_k \,. \tag{7}$$

The Stolper-Samuelson ordering from the 2×2 model reflects the factor intensity in (1),

$$\hat{w}_1 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2 \,. \tag{8}$$

Regions i, ii, and iii are first considered explicity by Jones and Easton. Remember that $\hat{w}_3 = 0$. The following five magnification effects (MEs) occur in the regions

$$\begin{array}{ll} \text{i} & \hat{w}_1 > \hat{p}_1 > \hat{w}_3 > \hat{p}_2 > \hat{w}_2 \\ \text{ii} & \hat{w}_1 > \hat{w}_3 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2 \\ \text{iii}_a & \hat{w}_3 > \hat{w}_1 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2 \\ \text{iii}_b & \hat{w}_3 > \hat{p}_1 > \hat{w}_1 > \hat{p}_2 > \hat{w}_2 \\ \text{iii}_c & \hat{w}_3 > \hat{p}_1 > \hat{p}_2 > \hat{w}_1 > \hat{w}_2 \end{array} \tag{9}$$

The numbering of MEs corresponds directly with the regions in Fig. 1. Moving from region i through ii to iii, the price of the middle factor 3 rises while the price of factor 1 sinks.

Consider a tariff on good 1, which is depicted by $\hat{p}_1 > \hat{p}_2 = 0$. Under this assumption, $\hat{w}_3 > 0$ in (9). The real income of extreme factor 1 rises under i, ii, and iii_a. The nominal wage of factor 1 rises in iii_b since $\hat{w}_1 > \hat{p}_2 = 0$, but the change in the real income of factor 1 depends on consumption shares. In iii_c, the nominal and real wage of factor 1 falls. The price of middle factor 3 rises everywhere in (9), outranking $\hat{p}_1$ except in region i. Extreme factor 2 suffers a falling price and

Since $\hat{w}_2 < 0$ in (12), it must be that $\hat{p}_2 > \hat{w}_2$. If $\hat{w}_1 > \hat{w}_3$, either i or ii is possible. If $\hat{w}_3 > \hat{w}_1$, iii_a and iii_b can be derived.

In b′, the first column of $\hat{w}_i/\hat{p}_1$ signs and the corresponding magnification effects in (10) are

$$[+ \; - \; -]' \qquad \text{iv}, \text{v}_a \, . \tag{13}$$

In sign patterns c′ and d′ from (3), the correspondence between the first column of ω and MEs in (10) and (11) is

$$[+ \; + \; -]' \qquad \text{v}_b, \text{v}_c, \text{vi} \, . \tag{14}$$

There is another ME which can be derived when $\hat{p}_1 > \hat{p}_2 = 0$ in sign patterns c′ and d′, namely

$$\hat{w}_2 > \hat{p}_1 > \hat{w}_1 > \hat{p}_2 > \hat{w}_3 \, . \tag{15}$$

Note that w_2 has risen above $\hat{p}_1$, partly reversing the Stolper-Samuelson ordering in (4) from the 2×2 model. It turns out, however, that (15) cannot be derived from the SPs when both columns are included in the analysis.

From d, two more possible ME rankings in (9) and (12) are

$$[- \; - \; +]' \qquad \text{iii}_c, \text{vii} \, . \tag{16}$$

To maintain the relative price changes in the second column of ω in (2), suppose $\hat{p}_1 = 0 > \hat{p}_2$. There are four patterns in the second column of the ω matrices in (3), leading to corresponding MEs,

$$[- \; + \; +]' \qquad \text{i}, \text{iv}, \text{v}_a, \text{v}_b \tag{17}$$

$$[- \; + \; -]' \qquad \text{ii}, \text{iii}_a \tag{18}$$

$$[+ \; + \; -]' \qquad \text{iii}_b, \text{iii}_c, \text{vii}, \hat{w}_3 > \hat{p}_1, \hat{w}_2 > \hat{p}_2 > \hat{w}_1 \tag{19}$$

$$[- \; - \; +]' \qquad \text{v}_c, \text{v} \, . \tag{20}$$

The eleven possible SPs are constructed directly from combinations of the first columns in (12), (13), (14), and (16), and the second columns in (17) through (20). An ME corresponding to a given SP must be consistent with both columns of the SP. For instance, in a, the first column $[+ \; - \; +]'$ in (12) combined with the second column $[- \; + \; +]'$ in (17) yields only one common ME, namely i. The MEs ii, ii_a, and iii_b are consistent with the first column of sign pattern a, but not with the second column. Likewise, iv, v_a, and v_b are consitent with the second column, but not the first.

The first column $[+ \; - \; +]'$ in (12) combined with the second column $[- \; + \; -]'$ in (18) leads to sign pattern b and yields two consistent MEs, ii and iii_a. The complete correspondence from SPs to MEs is

$$\begin{aligned} &a \rightarrow i \\ &b \rightarrow ii, iii_a \\ &b' \rightarrow iv, v_a \\ &c \rightarrow iii_b \\ &c' \rightarrow v_b \\ &d \rightarrow iii_c, vii \\ &d' \rightarrow v_c, vi\,. \end{aligned} \tag{21}$$

Note that the reversed Stolper-Samuelson ordering $\hat{w}_2 > \hat{p}_1 > \hat{p}_2 > \hat{w}_1$ is not derived. Supposing this reversed ordering holds leads to a direct contradiction. Since $\hat{p}_1 > \hat{p}_2$, it follows that $\hat{w}_1 > -(\theta_2 \hat{w}_2 + \theta_3 \hat{w}_3)/\theta_1$, where $\theta_i \equiv \theta_{i2} - \theta_{i1}$. Also, $\hat{p}_2 > \hat{w}_1$ implies $(\theta_{22}\hat{w}_2 + \theta_{32}\hat{w}_3)/\phi_{12} > \hat{w}_1$, where $\phi_{ij} \equiv 1 - \theta_{ij}$. Combining these two inequalities, $(\theta_{31}\phi_{12} - \theta_{32}\phi_{11})\hat{w}_3 > (\theta_{22}\phi_{11} - \theta_{21}\phi_{12})\hat{w}_2$. With $\sum_i \theta_{ij} = 1$ and $\theta_{22}\theta_{31} > \theta_{21}\theta_{32}$, it must be that $\hat{w}_3 > \hat{w}_2$. Similarly, $\hat{w}_2 > \hat{p}_1$ implies $(\theta_{11}\hat{w}_1 + \theta_{31}\hat{w}_3)/\phi_{21} < \hat{w}_2$. With $\hat{p}_1 > \hat{p}_2$, $\hat{w}_2 < -(\theta_1 \hat{w}_1 + \theta_3 \hat{w}_3)/\theta_2$. Combining these results in a similar fashion implies an inconsistent result, $\hat{w}_1 > \hat{w}_3$.

Given only the restriction in (7), there would be 18 seemingly possible MEs. Five of these, however, involve the reversed Stolper-Samuelson ordering. The other two MEs reled out by combining the columns of SPs arise in (15) and (19):

$$\begin{aligned} &\hat{w}_2 > \hat{p}_1 > \hat{w}_1 > \hat{p}_2 > \hat{w}_3 \\ &\hat{w}_3 > \hat{p}_1 > \hat{w}_2 > \hat{p}_2 > \hat{w}_1 \end{aligned} \tag{22}$$

These two MEs involve a "partial" reversal of the Stolper-Samuelson ordering in (8). If the percentage change in the price of an extreme factor moves to the opposite end of the magnification raking, it must keep the other extreme factor price with it, as in iii_c and v_c. The percentage change in the price of the other extreme factor cannot slide back between price changes.

The seven SPs in (3) thus lead to the complete set of eleven MEs, as in (21). If $\hat{p}_2 > \hat{p}_1$, an analogous set of MEs could be derived.

5. CONCLUSION

This note develops the relationship between comparative static sign patterns and magnification effects in the 3×2 general equilibrium model of production and trade. Eleven different magnification effects are isolated. A reversed Stolper-Samuleson result, stated in terms of the extreme factors, is impossible. Even a partial reversal cannot occur.

The fact that there is such a wide range of magnification effects is disquieting to theorists looking for simple generalities. If adding one productive factor causes the number of magnification effects to rise from one to eleven, additional goods or factors are likely to increase possibilities beyond reasonable inventory. Still, the fact that strict reversals cannot occur is a meaningful restriction.

In any applied situation, information on factor intensity and factor substitution would be used to limit the range of potential magnification effects. Descriptions of the internal workings of a particular economy leading to its comparative static properties could also be developed.

The magnification effect is a powerful conceptual device which should be generalized as much as possible. As economies continue to adjust to changing world markets or impose protection, price changes will redistribute income among productive factors.

Auburn University

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