A Note on the Oil Price Trend and GARCH Shocks

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This paper investigates the trend in the monthly real price of oil between 1990 and 2008 with a generalized autoregressive conditional heteroskedasticity (GARCH) model. Trend and volatility are estimated jointly with the maximum likelihood estimation. There is long persistence in the variance of oil price shocks, and a GARCH unit root (GUR) test can potentially yield a significant power gain relative to the augmented Dickey-Fuller (ADF) test. After allowing for nonlinearity, the evidence supports a deterministic trend in the price of oil. The deterministic trend implies that influence of a price shock is transitory and policy efforts to restore a predictable price after a shock would be unwarranted in the long run.

1. INTRODUCTION

From the perspective of both oil supply and demand, it is critical to understand price behavior following a shock. If the price were mean reverting or trend reverting, shocks would dissipate and policy efforts to restore price following a shock would be unwarranted. If, however, there were no price reversion in a random walk or a stochastic trend, policy intervention would be wise to overcome the permanent effect of a price shock.

For instance, price controls imposed by the US government following the oil price shock of the early 1970s would have been warranted if price were not to revert to its long term trend. The same can be said for the OPEC production quotas of the early 1980s. If price were a deterministic trend, however, the shocks would have had no permanent effects and these policies would have been redundant.

The present paper examines the evidence of a deterministic trend in the monthly real price of oil from 1990 to 2008, a period of relatively stable market

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structure with no such overwhelming breaks in the price series. Results from a GARCH unit root test that simultaneously estimates trend and variance are compared with well known difference stationarity tests. The present results have direct implications for energy policy.

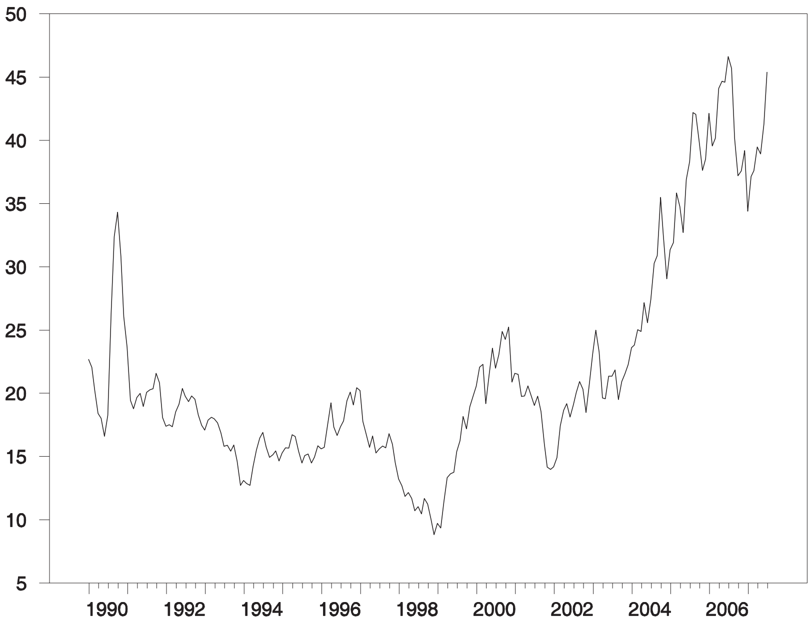
2. GARCH AND FOURIER METHODOLOGY

Suppose the price of oil p_t follows an AR(1) process $p_t = \beta_0 + \beta_1 p_{t-1} + e_t$. If $|\beta_1| < 1$ then $\partial p_{t+s}/\partial e_t = \beta_1^s \rightarrow 0$ as $s \rightarrow \infty$ and the effect of a shock diminishes with time. The series is then stationary and mean reverting following a shock. In contrast, if $\beta_1 = 1$ the effect of a shock never dies and the series is a nonstationary random walk with no reverting behavior.

Figure 1 presents the deflated monthly spot price of West Texas Intermediate Oil between January 1990 and February 2008 from the Federal Reserve Economic Data. The base year is 1990 and CPI is used for deflating nominal oil price. The issue is whether the price trend is deterministic or stochastic.

The present study focuses on monthly prices since most trading is done on a monthly basis. Also, the general autoregressive conditional heteroskedasticity(GARCH) effect becomes stronger and the GARCH unit root (GUR) test brings more power gain for higher frequency data. Preliminary analysis indicates that annual data lead to a weak GARCH effect and to difficulty in convergence of

Figure 1. Real Oil Price



the maximum likelihood estimator (MLE) algorithm. The dollar price is chosen following convention in the oil market and given the lack of significant swings in the trade weighted dollar.

Results of recent studies based on annual data are mixed. Berck and Roberts (1996) and Ahrens and Sharma (1997) find stochastic trends across a menu of unit root tests. Allowing for multiple breaks, Lee, List, and Strazicich (2006) report a deterministic trend. The present paper improves upon the previous methodologies in two ways.

First, there is evidence that monthly price shocks follow the GARCH process proposed by Bollerslev (1986) regardless of the specification of deterministic terms in mean regressions. Trend and volatility are then investigated jointly with an MLE-based GARCH unit root test. Seo (1999) shows that this GUR test utilizes information in the volatility and therefore enjoys a power gain relative to the augmented Dickey Fuller (ADF) test.

Another innovation of the present paper is the treatment of possible structural breaks. Breaks are accounted for first by dummy variables following Ahrens and Sharma (1997) and Lee, List, and Strazicich (2006). The dummy variable approach, however, is restrictive given the assumptions about the maximum number and the functional form of breaks. The present paper employs the Fourier form of Becker, Enders, and Hurn (2004) and Becker, Enders, and Lee (2006) that approximates breaks, or general nonlinear deterministic terms, with robustness.

3. MODEL SPECIFICATION FOR THE PRICE OF OIL

Consider solving the dynamic problem of extracting oil over n-periods subject to a total reserve constraint

$$\max_{q_t} \sum_{t=1}^{n} (1+r)^{-(t-1)} (k_1 q_t - k_2 q_t^2/2 - k_3 q_t) + \theta (Q - \sum_{t=1}^{n} q_t),$$

where q_t is the amount of oil extracted, r is the discount rate, $k_1 q_t - k_2 q_t^2/2$ is consumer surplus, k_3 is constant marginal cost, θ is the Lagrangian multiplier, and Q is oil reserves. First order conditions imply $(1 + r)^{-(t-1)} (k_1 - k_2 q_t - k_3) - \theta = 0$ from which the price of oil $p_t = k_1 - k_2 q_t$ satisfies the first-order difference equation $p_t = (1 + r)p_{t-1} - rk_3$.

There are various ways to add a stochastic element to price, and both stochastic and deterministic trends can be accommodated by this extraction model. Consider first adding the stationary process e_t in the difference equation $p_t = (1 + r)p_{t-1} - rk_3 + e_t$. Price is then a random walk when $r = 0$, and this random walk has a drift term if the mean of e_t is not zero. The trend accumulated by the drift term is stochastic.

Alternatively, solve the difference equation to find $p_t = (1 + r)^t + c$. We have a trend stationary price of oil after adding a stationary process to the solution $p_t = (1 + r)^t + c + e_t$. In this case, the trend is deterministic and nonlinear.

Statistically we use the following models to test the type of trend.

$$\Delta p_t = \alpha_0 + \alpha_1 t + \beta_0 p_{t-1} + \sum_{j=1}^{m} \beta_i \Delta p_{t-i} + e_t \tag{1}$$

$$\Delta p_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \beta_0 p_{t-1} + \sum_{j=1}^{m} \beta_i \Delta p_{t-i} + e_t \tag{2}$$

$$\begin{aligned} \Delta p_t &= \alpha_0 + \alpha_1 t + \alpha_3 DU_{1t} + \alpha_4 DT_{1t} + \alpha_5 DU_{2t} \\ &\quad + \alpha_6 DT_{2t} + \beta_0 p_{t-1} + \sum_{j=1}^{m} \beta_i \Delta p_{t-i} + e_t \end{aligned} \tag{3}$$

$$\begin{aligned} \Delta p &= \alpha_0 + \alpha_1 t + \alpha_5 \sin\left(\frac{2\pi\omega t}{T}\right) + \alpha_6 \cos\left(\frac{2\pi\omega t}{T}\right) \\ &\quad + \beta_0 p_{t-1} + \sum_{j=1}^{m} \beta_i \Delta p_{t-i} + e_t \end{aligned} \tag{4}$$

Note the linear and quadratic trends are included in (1) and (2), respectively. Let T^{10} and T^{20} be the two unknown break dates. The dummy break variables in (3) are specified as $DU_{jt} = 1$ and $DT_{jt} = t - T^{j0}$ when $t > T^{j0}$, ($j = 1,2$) and 0 otherwise. Lee and Strazicich (2001) shows that the ADF type unit root tests that allow for endogenous structural breaks have the drawback of spurious rejection of the null hypothesis. By contrast the LM test proposed by Lee and Strazicich (2003) does not have that drawback, and therefore the LM test is used here. The Fourier form in (4) employs sin and cos terms to approximate instantaneous or gradual breaks in deterministic terms. The Fourier form may also describe a general nonlinear model without the necessity of breaks.[1]

The break dates and the frequency ω are estimated with data dependent methods. The two break dates are estimated by the values that minimize the LM statistics of Lee and Strazicich (2003). There is a possible efficient estimator of the break datethat minimizes the residual sum squares (RSS) weighted by the shock variance.[2] The estimated frequency minimizes RSS. In general, the distribution for the unit root test with unknown breaks or unknown Fourier frequency is different from that with known breaks or frequency. Critical values for the tests are simulated if unavailable in the literature.

The primary goal is to test the null hypothesis $H_0;\beta_0 = 0$ in (1) through (4). The GUR test explicitly takes into account information in the variance of e_t. In contrast, ADF tests do not include any effect of variance. The conditional variance of e_t is specified in the following GARCH process

$$\sigma_t^2 = c_0 + \sum_{i=1}^{p} c_i \sigma_{t-i}^2 + \sum_{i=1}^{q} d_i e_{t-i}^2 \tag{5}$$

1. We are grateful to a referee for pointing this out.
2. We thank a referee for suggesting this estimator.

where σ_t^2 denotes the conditional variance of e_t. Hillebrand (2005) shows long persistence in variance may be caused by breaks in the variance, a complexity left for future research since the related GUR test is underdeveloped.

Equations (1) through (4) along with (5) are estimated with the maximum likelihood method. Let β_0^{MLE} denote the MLE estimate of β_0. The GUR test is computed as

$$GUR = \frac{\beta_0^{MLE}}{Standard\ error\ of\ \beta_0^{MLE}} \tag{6}$$

Seo (1999) shows the null distribution of the GUR test is a weighted average of the Dickey Fuller t and standard normal distributions. The weight is controlled by the parameter ρ that is bounded between 0 and 1. A smaller ρ implies more weight for the standard normal distribution and more power gain from the GUR test.

4. RESULTS

Table 1 reports the main findings. The GUR test is justified by various deterministic terms in mean regressions since ARCH tests always indicate autoregressive conditional heteroskedasticity. The GUR weight ρ is close to 0.6 in all cases implying the Dickey-Fuller t distribution only accounts for 0.6/(0.6 +

Table 1. Results

Model	**(1)**	**(2)**	**(3)****	**(4)**
ARCH(4) test	14.50*	14.32*	22.82*	23.82*
ρ	0.55	0.57	0.57	0.58
ADF test	-0.98	-3.96*	na	-4.09*
LM test***	na	na	-6.12*	na
BIC[1]	1.34	1.29	1.34	1.34
GUR Test	-1.18	-3.95*	-6.30*	-5.04*
BIC[2]	1.49	1.44	1.47	1.44
a_1	0.0042*	-0.0430*	-0.0143*	0.3293*
a_2	na	0.0002*	na	na
a_3	na	na	1.6482*	na
a_4	na	na	0.0112	na
a_5	na	na	-0.2506	-55.228*
a_6	na	na	0.1411*	19.596

*Significant at 5% level. ARCH(4) denotes the test for autoregressive conditional heteroskedasticity with 4 lags. ρ denotes the weight of Dickey-Fuller distribution. BIC[1] denotes the BIC for the ADF or LM test, and BIC[2] for the GUR test. **The estimated break dates are July 1999 and December 2002. ***See Lee and Strazicich (2003) for the details about the LM unit root test with two breaks.

$(1 - 0.6^2)^{1/2}) = 43\%$ of the hybrid distribution.[3] The power gain of the GUR test is substantial.

The ADF test, LM test with two breaks, and GUR test reject a stochastic trend at the 5% level except for model (1), providing strong evidence against a stochastic trend. The sharp difference between (1) and (2) through (4) illustrates the importance of allowing for the nonlinear specification when examining the time series properties of oil price. Nonlinear specification is also emphasized by Ahrens and Sharma (1997) and Lee, List and Strazicich (2006).

Similar results from the ADF/LM tests and GUR test are not evidence against the GUR test. Notice that the ADF and LM tests reject the null hypothesis for models (2) through (4). That means the power of the ADF and LM tests is so high (for this problem) that it disables the presumably superior GUR test to produce qualitatively different results.

The BIC for the ADF/LM tests (denoted by BIC^1) and for the GUR test (denoted by BIC^2) are reported for model selection. The quadratic trend model (2) outperforms the linear trend model (1) by the BIC criterion as it is better able to capture the upturn in 1999. Two dummy break variables in (3) and the sin term of the Fourier model (4) are significant. The Fourier model (4) outperforms the dummy model (3) by BIC^2. Both BIC^1 and BIC^2 pick the quadratic trend model (2) as the best model.

The conclusion is that the trend in the price of oil is deterministic. The trend may be quadratic or linear with unspecified breaks, or it may have a general nonlinear functional form.

5. CONCLUSION

The deterministic trend in the monthly price of oil between 1990 and 2008 suggests policy reactions to oil price shocks are unwarranted in the long run since price reverts to its long term trend. Evidence of the deterministic trend also provides support for optimal depletion models of Hotelling (1931). Our main findings are consistent with those of Lee, List, and Strazicich (2006). In terms of methodology, the findings in this paper contribute to the growing literature that suggests price models can be improved by allowing for a nonlinear specification.

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3. See Equation (13) of Seo (1999) for details.

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