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Robustness of the Stolper–Samuelson Intensity Price Link

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Chapter Outline

The Stolper–Samuelson theorem isolates conditions under which factor intensity determines the qualitative factor price adjustments to price changes in general equilibrium. The present chapter examines the robustness of this "intensity price link" under relaxations of its sufficient conditions, with parametric specifications of the comparative static model based on neoclassical production, competitive pricing, and full employment.

1 Robustness of the Stolper–Samuelson Intensity Price Link

The Stolper–Samuelson (1941) theorem isolates a set of conditions under which factor intensity is sufficient to determine the qualitative effects of price changes on factor prices. Its novel property is that factor substitution plays no role. A literature evolved pointing out that the theorem does not hold under other conditions, implicitly suggesting a limited scope. The present chapter points out, however, that the Stolper–Samuelson intensity price link is generally robust to parametric relaxations of its sufficient conditions. The scope of the theorem is widened as it is shown to hold under much wider initial conditions than suggested by the list of sufficient conditions. None of the sufficient conditions are necessary for the intensity price link.

The next section reviews the proof of the Stolper–Samuelson theorem. The following sections analyze the intensity price link assuming in turn international factor mobility, nontraded products, factor intensity reversals, elastic factor supply, unemployment, factor market distortions, noncompetitive pricing of outputs, increasing returns, and nonhomothetic production. The intensity price link may hold under any of these conditions and when it is relaxed it is only partly so. Increasing returns are analyzed with a general cost function revealing new patterns of factor price adjustments. A final section summarizes models with many factors and many products, including a high dimensional measure of factor intensity.

2 PROOFS OF THE INTENSITY PRICE LINK

Proofs of the Stolper–Samuelson theorem follow the work of Koo (1953), Jones (1956), Lancaster (1957), Bhagwati (1959), and Chipman (1966). Its sufficient assumptions include:

- two homogeneous traded products in a small open economy;
- two homogeneous factors, mobile nationally but immobile internationally;
- perfect competition in product and factor markets;
- perfectly inelastic factor supply;
- full employment; and
- linearly homogeneous production functions.

The following sections relax these assumptions using parametric modifications of the algebraic comparative static model. The linearly homogeneous assumption is relaxed with both variable returns and nonhomothetic production. There are other implicit underlying assumptions, including the absence of specific factors, joint production, intermediate products, depletable or renewable resources, and production of capital goods.

The starting point is a 2×2 production box, explaining in part the enduring pedagogical popularity of the theorem. Along the contract curve, suppose factor 1 is used intensively in product 1,

$$v_{11}/v_{21} > v_{12}/v_{22} \tag{3.1}$$

where v_{ij} is the input of factor i in the production of product j, $i,j = 1,2$. The contract curve does not cross the diagonal because with homothetic production if a point on the diagonal were on the contract curve all points would have to be. While there can be no factor intensity reversals due to price changes in the economy with linearly homogeneous production, with three factors there could be.

Each endogenous factor price w_i is equal across sectors in the economy and isoquants of the two sectors share a common tangency and the same relative factor price. Exogenous prices p_j for the two traded products determine output levels and corresponding relative factor prices along the contract curve. A higher relative price for product 1 would raise its output and the relative price w_1/w_2 of its intensive

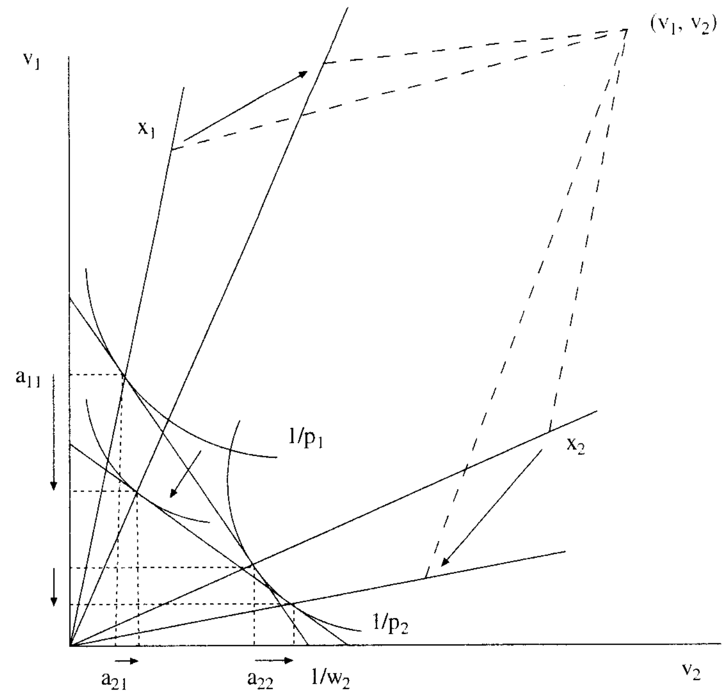

Figure 3.1 Stolper-Samuelson adjustment

factor. Input ratios v_{ij}/v_{2j} would fall in each sector as cost minimizing firms adjust to the new higher relative price of factor 1.

Figure 3.1 presents the corresponding 2 × 2 Lerner-Pearce production diagram. Unit value isoquants $x_j = 1/p_j$ represent the amount of each product worth one unit of numeraire. If dollars are the numeraire, it follows that p_j = \$/product and $1/p_j$ = product/\$. Neoclassical production functions imply concave isoquants with positions of unit value isoquants determined by exogenous prices p_j in the small open economy.

The unique unit isocost line $c_j = 1 = a_{ij}w_1 + a_{2j}w_2$ shows input combinations that cost \$1 and supports the unit isoquants due to cost minimization. Endpoints of the unit value isocost line are $1/w_i$. Firms minimize cost $c_j = \Sigma_i a_{ij}w_i$ where a_{ij} is the cost minimizing amount of factor i used in the production of a unit of product j. Competition ensures $p_j = c_j$, uniquely determining the endogenous w_i at the endpoints of the unit isocost line. The endogenous $a_{ij}(w)$ are functions of the vector of endogenous factor prices w. The factor intensity condition in (3.1) can be stated in terms of relative inputs,

$$a_{11}/a_{21} > a_{12}/a_{22}. \tag{3.2}$$

as reflected by the steeper expansion path for sector 1.

In Figure 3.1, a ceteris paribus increase in p_1 shifts that unit value isoquant toward the origin as one dollar's worth becomes less of the physical product. The isocost line rotates around isoquant 2, the price of intensive factor w_1 rising while w_2 falls. Production becomes more intensive in relatively cheaper factor 2 as a_{1j} falls and a_{2j} rises. In the matrix of $\delta w_i/\delta p_j \equiv w_{ij}$ results, there is a positive main diagonal with negative elements off the diagonal.

$$\begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{pmatrix} = \begin{pmatrix} + & - \\ - & + \end{pmatrix} \tag{3.3}$$

The algebraic general equilibrium model will be used to introduce parametric relaxations of the various assumptions. Chipman (1966) and Takayama (1982) present the foundations of full employment of factors and competitive pricing of products. Full employment for factor i is stated $v_i = \Sigma_j a_{ij} x_j$ where v_i is the endowment of factor i and x_j the output of product j. Differentiate to find $dv_i = \Sigma_j a_{ij} dx_j + \Sigma_j x_j da_{ij}$. With homothetic production, cost minimizing unit inputs a_{ij} are functions of factor prices alone and $da_{ij} = \Sigma_k(\delta a_{ij}/\delta w_k)dw_k$. It follows that $\Sigma_j x_j da_{ij} = \Sigma_k(\Sigma_j x_j \delta a_{ij}/\delta w_k)dw_k = \Sigma_k s_{ik} dw_k$ given the output weighted substitution term $s_{ik} \equiv \Sigma_j x_j \delta a_{ij}/\delta w_k$. Shephard's lemma states that cost minimizing inputs are partial derivatives of cost functions, $a_{ij} = \delta c_j/\delta w_i$ and it follows that $\delta a_{ij}/\delta w_k = \delta^2 c_j/\delta w_i \delta w_k$. Young's theorem on the symmetry of partial derivatives then implies $s_{ik} = s_{ki}$. For notation, $s \equiv s_{12} = s_{21}$. Own substitution terms s_{ii} are negative due to concavity of cost functions. Summing across weighted substitution terms, $\Sigma_i w_i s_{ik} = \Sigma_i w_i \Sigma_j x_j(\delta a_{ij}/\delta w_k) = \Sigma_j x_j \Sigma_i w_i(\delta a_{ij}/\delta w_k) = \Sigma_j x_j \Sigma_i w_i(\delta a_{kj}/\delta w_i) = 0$ by Euler's theorem. Without loss of generality, rescale factors so $w_i = 1$ and it follows that $s = -s_{11} = -s_{22} = s_{12} = s_{21}$. Full employment is stated in the first two equations of the comparative static system (3.4) below.

Competitive pricing for product j is stated $p_j = \Sigma_i a_{ij} w_i$. Differentiate to find $dp_j = \Sigma_i a_{ij} dw_i + \Sigma_i w_i da_{ij}$. Firms minimize cost, implying the slope of each unit value isoquant da_{1j}/da_{2j} equals the slope of the isocost line $-w_2/w_1$. The cost minimizing envelope $\Sigma_i w_i da_{ij} = 0$ follows, implying $dp_j = \Sigma_i a_{ij} dw_i$. Competitive pricing is stated in the second two equations of the 2×2 comparative static factor proportions model,

$$\begin{pmatrix} -s & s & a_{11} & a_{12} \\ s & -s & a_{21} & a_{22} \\ a_{11} & a_{21} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} dw_1 \\ dw_2 \\ dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} dv_1 \\ dv_2 \\ dp_1 \\ dp_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ dp_1 \\ dp_2 \end{pmatrix} \tag{3.4}$$

Factor endowments are held constant, $dv_i = 0$. The factor intensity condition in (3.2) implies $a_{11}a_{22} - a_{12}a_{21} \equiv b > 0$. The positive determinant in (3.4) is $\Delta = b^2$. Factor price equalization occurs inside the production cone of McKenzie (1955) where $\delta w_i/\delta v_k = 0$.

The $\delta w_i/\delta p_j$ or w_{ij} terms are derived from cofactors in the lower left partition of the system matrix using Cramer's rule,

$$\begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{pmatrix} = \begin{pmatrix} a_{22}/b & -a_{12}/b \\ -a_{21}/b & a_{11}/b \end{pmatrix}, \tag{3.5}$$

confirming the intensity price link in (3.3). Note that factor substitution has no effect on the w_{ij} terms. It is a surprise that factors might be perfect substitutes or not substitutes at all and the w_{ij} terms would be identical, a peculiar result that holds for "even" models with the same number of factors and products.

Jones (1965) develops the magnification effect that price changes are weighted averages of factor price changes. In the differentiated competitive pricing condition for product j, $dp_j = \Sigma_i a_{ij} dw_i$, divide both sides by p_j and multiply the left side by w_i/w_i to find $\Sigma_i \theta_{ij} \hat{w}_i = \hat{p}_j$, where ^ represents percentage change and $\theta_{ij} = w_i a_{ij}/p_j$, a factor share. In even models, the intensity price link in elasticity form is determined by properties of the θ matrix in $\theta\hat{w} = \hat{p}$ since $\hat{w}/\hat{p} = \theta^{-1}$. Percentage price changes are weighted averages of factor price changes. In the 2×2 model, if $\hat{p}_1 > \hat{p}_2$ it must be that $\hat{w}_1 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2$. If a single price increases, at least one factor price must rise more in percentage terms and the other factor price falls. For any nonzero vector of price changes, the real income of one factor must rise and the other must fall.

3 International Mobility of Factors and Nontraded Products

Adding internationally mobile factors of production or nontraded products may leave the intensity price link intact. As an example, consider the 3×2 model. The three factors capital, labor, and land provide the foundation for classical economics. Branson and Monoyios (1977) and Thompson (1997b) provide some motivation for trade models with separate skilled and unskilled labor. Batra and Casas (1976), Ruffin (1981), Takayama (1982), Suzuki (1983), Jones and Easton (1983), and Thompson (1985) develop theoretical properties of the 3×2 model. Factors can be unambiguously ranked according to factor intensity,

$$a_{11}/a_{12} > a_{21}/a_{22} > a_{31}/a_{32}. \tag{3.6}$$

Factor 1 is the extreme factor for product 1, factor 3 is extreme for product 2, and factor 2 is the middle factor. Thompson (1985) uncovers the possible sign patterns of w_{ij} terms, which depend on factor intensity as well as factor substitution. Isolating the two extreme factors, the possible sign patterns are

$$\begin{pmatrix} w_{11} & w_{31} \\ w_{12} & w_{32} \end{pmatrix} = \begin{matrix} + & - \\ - & + \\ (a) \end{matrix} \quad \begin{matrix} + & + \\ - & + \\ (b) \end{matrix} \quad \begin{matrix} + & + \\ - & - \\ (c) \end{matrix} \tag{3.7}$$

Sign pattern (3.7a) is the strong result, analogous to the 2×2 model. A higher price of product 1 unambiguously lowers the output of product 2 and demand for extreme factor 3 is expected to fall but in (3.7b) prices of both extreme factors rise. The expanding sector 1 can increase its input of factor 3, releasing complementary middle factor 2. In (3.7c), a higher price for product 2 also lowers the price of its extreme factor. Thompson (1986) isolates various conditions favoring factor price polarization, the separation of international factor prices with a move to free trade. The strong result in (3.7a) cannot be reversed completely as Thompson (1993) notes for the 3×2 magnification effect. Thompson (1995) uses sensitivity analysis in simulations of a 3×2 model of the US economy with skilled and unskilled labor and finds the intensity price link in (3.7a) due to the overwhelming influence of factor intensity.

Internationally mobile factors with factor prices exogenous at world levels can restore the intensity price link. If the middle factor in the 3×2 model is internationally mobile, there is the strong intensity price link in (3.7a). In the $r \times 2$ model when $r > 2$, the w_{ij} matrix has more than a single possible sign pattern and the intensity price link may break down. International mobility of $r - 2$ of the factors, however, would make factor prices exogenous and restore the intensity price link. In the $r \times 2$ model with $r - 2$ of the factors internationally mobile, there is an intensity price link for the internationally immobile factors.

With more products than factors in a small open economy, the comparative static system is overdetermined. Melvin (1968), Travis (1972), and Rader (1979) develop properties of the 2×3 and $2 \times n$ models, $n > 2$. Factor intensity can be unambiguously defined as a ranking of relative inputs across industries when there are two factors. When $n > r = 2$ in a small open economy, however, there are more than two arbitrarily placed unit value isoquants and for almost any set of world prices there is no unique supporting isocost line. Product prices may be assumed to adjust as in Choi (2003) but short of a solution algorithm little more can be said about the intensity price link.

Introducing nontraded products, however, endogenizes prices and can restore the intensity price link. Komiya (1967) and Rivera-Batiz (1982) develop models with nontraded products. In the $2 \times n$ model, if $n - 2$ of the products are nontraded there is a strong intensity price link for the traded products. In the 2×3 model with one nontraded product, Ethier (1972) examines conditions that lead to an intensity price link. Although demand conditions might relax the intensity price link, it is robust to "small" demand elasticities.

4 Factor Intensity Reversals and the Intensity Price Link

Production cones are regions in factor space between expansion paths where all products can be produced with full employment of all factors. Expansion paths are linear with homothetic production. Production cones are generally not unique. Even in the 2×2 model, there are two production cones if the isoquants cross twice. Pearce (1951), James and Pearce (1951), and Harrod (1958) make the point that factor price equalization would not occur with free trade if endowments of the two trading countries lie in different production cones. The country abundant in a factor would maintain a lower relative price for that factor with free trade. The intensity price link would nevertheless hold inside each country because the opposite factor is used intensively in each country. If sector 1 uses factor 1 intensively in country 1 but uses factor 2 intensively in country 2, a higher price for product 1 would raise w_1 in country 1 and w_2 in country 2. The intensity price link holds in each country and free trade raises the price of cheap abundant factors.

"Internal" factor intensity reversals can occur in an economy with more than two factors because changing prices can potentially lead to factor intensity reversals inside the country. In the 3×2 model, a factor might be the extreme factor in one sector but could lose that ranking due to some vector of price changes. Wong (1990) shows that such internal factor intensity reversals are impossible with exponential production functions although they may not be ruled out for homothetic production functions. With nonhomothetic production functions, internal factor intensity reversals certainly may occur. While the observation of an internal factor intensity reversal is consistent with nonhomothetic production, it may only point to the presence of more than two factors. An internal factor intensity reversal does not violate the intensity price link as much as it requires a careful statement of the changes taking place in the economy.

5 Elastic Factor Supply and the Intensity Price Link

Factors supplies are assumed to be perfectly inelastic in the factor proportions model but there is ample evidence that quantity supplied increases with price in some labor and natural resource markets. Kemp and Jones (1962) examine the effects of elastic factor supply on offer curves. Upward sloping factor supply can be included directly in the algebraic comparative static model. Suppose the supply of factor 1 is a positive function of its own price, $v_1(w_1)$, where $dv_1/dw_1 \equiv v_{11} > 0$. In the comparative static model (3.4), the first equation becomes $(s_{11} - v_{11})dw_1 + s_{12}dw_2 + a_{11}dx_1 + a_{12}dx_2 = 0$ and the qualitative intensity price link could be affected by the elasticity of factor supply v_{11}.

Note, however, that substitution plays no role in the Stolper–Samuelson w_{ij} terms in (3.5) and that factor price equalization implies there would be no effect of the induced change in v_1 (w_1) on factor prices. Factor prices are insulated from induced factors supply changes if factor price equalization holds. In models without factor price equalization, factor prices would vary with an induced change in factor supply. The own effects of factor endowments on factor prices are negative, $\delta w_i/\delta v_i < 0$, implying factor demands slope downward in the general equilibrium. As an example, consider the 3×2 model and suppose $\delta w_1/\delta p_1$ would be positive in the absence of the induced effect on the supply of factor 1. An increase in w_1 induces an increase in v_1 dampening the increase in w_1. If v_{11} is large enough, the positive sign of $\delta w_1/\delta p_1$ is reversed suggesting elastic factor supply could alter the intensity price link. Nevertheless, factor intensity would remain a fundamental determinant of the w_{ij} sign pattern and would have to be overcome by a strong factor supply effect. The intensity price link would hold for a range of factor supply elasticities.

6 Unemployment and the Intensity Price Link

Unemployment can arise for various reasons and there is a literature that bridges international and labor economics. The ultimate effect of introducing unemployment is that the wage does not fall to clear the labor market. In the general equilibrium model of Thompson (1989) with the unemployment rate varying endogenously and inversely with aggregate output, the intensity price link is unaffected.

In factor proportion models, unemployment occurs when the quantity of labor demanded falls short of the inelastic quantity supplied. Full employment of factor k can be stated $v_k = D_k(\mathbf{p}, \mathbf{v})$ where the quantity of labor demanded in the general equilibrium $D_k(\mathbf{p}, \mathbf{v})$ is a function of the vectors of exogenous variables $\mathbf{p}$ and $\mathbf{v}$. There is unemployment if $D_k(\mathbf{p}, \mathbf{v}) < v_k$ at the current wage. The general equilibrium effects of increased unemployment would be the same as a reduction in the labor endowment since the economy employs less labor.

In the 2×2 model, increased unemployment would affect outputs but because of factor price equalization factor prices and the intensity price link are not affected. The level of employment does not affect factor prices as long as employment remains inside the production cone, a principle in any model with factor price equalization. In models without factor price equalization, however, a change in the unemployment rate affects factor prices. An increased unemployment rate may involve a higher wage but some other factor prices would have to fall in the absence of factor price equalization, similar to the effects of a change in a factor endowment.

Thompson (1997) points out that the $\delta w_i/\delta v_k$ results are apparently nearly zero when they are not zero. This "near factor price equalization" suggests that changes in unemployment would generally have negligible impacts on factor prices and the intensity price link.

Turning briefly to a parameterized model, consider unemployment in the market for factor 1 with $\beta v_1 = \Sigma_j a_{1j} x_j$ where $\beta < 1$. If β is constant, the intensity price link in the comparative static w_{ij} terms is unaffected. The entire adjustment process in the factor markets is forced onto factor prices. To introduce flexibility, let β be a negative function of the factor price, $\beta(w_1)$ with $\beta' < 0$. The first equation in the comparative static system (3.4) becomes $(s_{11} - v_1\beta')\mathrm{d}w_1 + s_{12}\mathrm{d}w_2 + \Sigma_j a_{1j}\mathrm{d}x_j = \beta\mathrm{d}v_1 = 0$. The w_{ij} results in (3.5) are unaffected and the strong intensity price link holds. The w_{11} term is dampened by unemployment but if positive cannot switch signs. The higher w_1 due to an increase in p_1 lowers β, dampening the increase in w_1.

Unemployment has the potential to change the factor intensity of employed factors and alter interpretation of the intensity price link. Nevertheless, full employment is not a necessary condition for the factor intensity price link as the present parameterized model shows.

7 Factor Market Distortions and the Intensity Price Link

The factor market distortions in the present section cause a factor price to be different across sectors. Taxes, unionization, minimum wages, location, and different working conditions can lead to such distortions. Johnson (1966), Johnson and Mieszkowski (1970), Jones (1971), Herberg and Kemp (1971), Magee (1971, 1973), and Bhagwati and Srinivasan (1971) introduce such distortions into the factor proportions model. The present section considers the robustness of the intensity price link in the presence of a parametric distortion in the intersector market for factor 1.

Let w_1^s be the price of factor 1 in sector s and suppose $\gamma w_1^1 = w_1^2$. If $\gamma = 1$ there is no factor market distortion. Consider the situation where $\gamma > 1$ and factor 1 receives a premium in sector 2. A change in w_1^2 would be written $dw_1^2 = \gamma dw_1^1 + w_1^1 d\gamma$. For simplicity, assume the premium is constant in the comparative statics, $d\gamma = 0$. A change in the price of product 2 is then $dp_2 = a_{12}dw_1^2 + a_{22}dw_2 = a_{12}\gamma dw_1^1 + a_{22}dw_2$. In sector 1, $dp_1 = a_{11}dw_1^1 + a_{21}dw_2$. Substitution terms have to be recalculated and are represented by s_d. The comparative static model with a factor price premium:

$$\begin{pmatrix} -s_d & s_d & a_{11} & a_{12} \\ s_d & -s_d & a_{21} & a_{22} \\ a_{11} & a_{21} & 0 & 0 \\ a_{12}\gamma & a_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} dw_1^1 \\ dw_2 \\ dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} dv_1 \\ dv_2 \\ dp_1 \\ dp_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ dp_1 \\ dp_2 \end{pmatrix} \tag{3.8}$$

has determinant $\Delta_d = bb_\gamma$ where $b_\gamma \equiv (a_{11}a_{22} - \gamma a_{12}a_{21})$. In the undistorted model where $\gamma = 1$, the positive determinant is b^2 and the intensity price link in (3.5) emerges. If $\gamma > 1$, however, the signs of b_γ and Δ_d are ambiguous. The w_{ij} results are

$$\begin{pmatrix} w_{11}^1 & w_{21} \\ w_{12}^1 & w_{22} \end{pmatrix} = \begin{pmatrix} a_{22}/b_\gamma & -a_{12}\gamma/b_\gamma \\ -a_{21}/b_\gamma & a_{11}/b_\gamma \end{pmatrix} = \begin{matrix} + & - \\ - & + \end{matrix} \quad \begin{matrix} - & + \\ + & - \end{matrix} \tag{3.9}$$

Related effects on the price of factor 1 in sector 2 are $w_{11}^2 \equiv \delta w_1^2/\delta p_1 = \gamma(\delta w_1^1/\delta p_1) = \gamma/b_\gamma$ and $w_{12}^2 \equiv \delta w_1^2/\delta p_2 = \gamma(\delta w_1^1/\delta p_2) = -\gamma^2/b_\gamma$. If $b_\gamma > 0$ the strong intensity price link holds but if $b_\gamma < 0$ it is reversed. If γ is large enough to make b_γ negative, factor intensity is effectively reversed making factor 1 intensive in sector 2 and reversing the intensity price link. These results would not necessarily change if γ were an endogenous function of other variables in the model. If γ starts at unit value and $b > 0$, letting γ increase will decrease b_γ, increasing the sizes of the w_{ij} terms. As b_γ approaches zero the model becomes unstable and the w_{ij} terms explode. When b_γ becomes negative the w_{ij} terms switch signs although there is instability in the neighborhood where $\gamma = a_{11}a_{22}/a_{12}a_{21}$. The important point for the present purpose is that the presence of a factor market distortion does not necessarily relax the intensity price link.

8 Noncompetitive Pricing and the Intensity Price Link

Competitive pricing of products is another sufficient condition for the Stolper–Samuelson theorem. Models of production and trade for small open economies can be closed without a utility structure if competitive firms produce where cost equals the exogenous world price. Melvin and Warne (1973) examine monopoly pricing in the context of utility maximization. In models of monopolistic competition such as Krugman (1979) and Helpman (1981) demand is introduced and pricing remains competitive. Wong (1995, chapter 7) examines an international duopoly with products produced by single firms in each of two countries and finds the Stolper–Samuelson theorem may hold. Melvin and Warne (1973) make the same point when both sectors are international duopolists colluding to maximize joint profit. Kemp and Okawa (1998a, b) show the intensity price link is robust when oligopolists are a primary factor paid profit.

The present section introduces a wedge parameter between price and cost in a small open economy. Suppose there is a monopoly in sector 1 based on ownership of a natural resource or another legal entry restriction. Such a monopoly is a price taker in the international market but searches for the output that maximizes profit. Raising monopoly output increases cost in the general equilibrium by raising relative demand for its intensive factor, but revenue also increases. The monopoly has some monopsony power over the factor markets in the small open economy.

Competitive pricing implies a tangency between the unit value isoquant and the unit isocost line as in figure 3.1. If cost were less than price in sector 1 due to monopoly power, the unit isocost line would instead cut through the unit value isoquant. Given that the monopoly minimizes cost, the input ratio would be determined by the tangency of an isocost line with the c_1 isoquant that represents the amount of the product that costs one unit to produce at current factor prices. Product 1 is sold at a price higher than cost as pictured in figure 3.2. With restricted output, the relative price of factor 1 would be lower and the relative input of factor 1 higher than with competitive pricing.

Profit of the monopolist is $\pi_1 = (p_1 - c_1)x_1$. Maximizing π_1 with respect to c_1, $0 = \delta\pi_1/\delta c_1 = (p_1 - c_1)(\delta x_1/\delta c_1) - x_1$. As an alternative, the monopolist in Thompson (2002) maximizes profit with respect to output. The term $\delta x_1/\delta c_1$ in the general equilibrium is the same as $\delta x_1/\delta p_1 = c^2s/b^2$ from the competitive model in (3.4) where $c \equiv a_{12} + a_{22}$. When the monopoly restricts output, the cost reducing effect is similar to an exogenous decrease in p_1 in the competitive model (3.4). Substituting and solving for the optimal level of cost, $c_1^* = p_1 - (b^2x_1/c^2s)$ which implies $c_1^* < p_1$. A profit maximization is implied because $\delta^2\pi_1/\delta c_1^2 = (p_1 - c_1)(\delta^2x_1/\delta^2c_1) - \delta x_1/\delta c_1 - x_1 < 0$ given $\delta x_1/\delta c_1 = \delta x_1/\delta p_1 = c^2s/b^2$ and $\delta^2x_1/\delta^2p_1 = 0$. The relationship between p_1 and c_1 is summarized by $p_1 = \alpha c_1$, where $\alpha \geq 1$. If there is competitive pricing, $\alpha = 1$. Substituting the optimal c_1^*, $\alpha^* = p_1/(p_1 - (b^2x_1/c^2s)) = c^2sp_1/(c^2sp_1 - b^2x_1) > 1$.

The monopoly profit margin may be regulated to maintain cost at a constant proportion of price, $p_1 = \alpha c_1$ where $\alpha > 1$. The monopoly would restrict output to α^* but a regulator might set α above α^*. Profit would then be proportional to revenue,

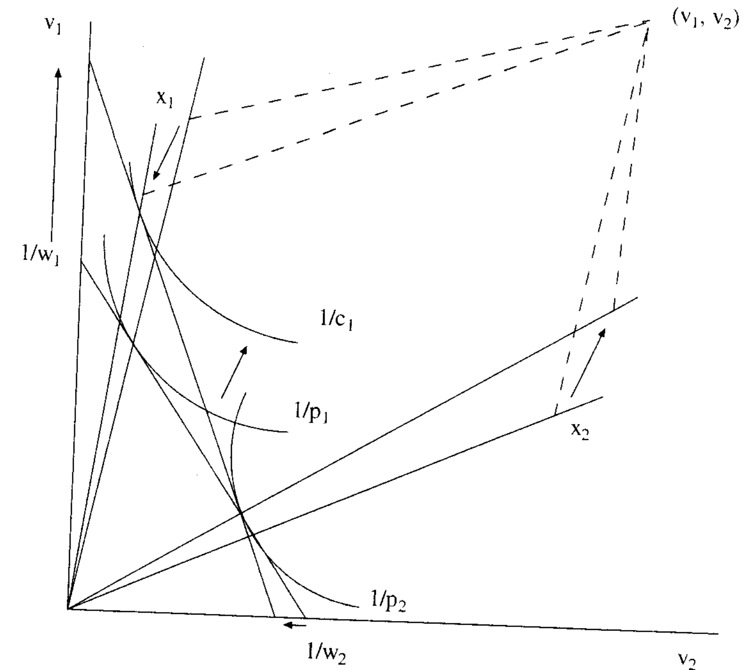

Figure 3.2 Monopoly output restriction

$\pi_1 = (1 - \alpha^{-1})p_1x_1 > 0$. With α constant, $dp_1 = \alpha dc_1 = \alpha\Sigma_i a_{i1}dw_i$ due to cost minimization along the monopoly unit cost isoquant, replacing the third equation in (3.4) and the effects of a change in the price of the competitive product 2 are identical to the competitive model (3.5). The factor price effects of a change in p_1 on the monopoly comparative static system are $w_{11} = a_{22}/\alpha b$ and $w_{21} = -a_{12}/\alpha b$, smaller than with competitive pricing. Changes in the price of the monopoly product have dampened Stolper–Samuelson effects relative to competitive pricing because of the corresponding proportional cost adjustment. The intensity price link, however, is robust with respect to this monopoly power.

Suppose the profit margin varies with the level of output: $\alpha(x_1)$. The optimal α^* increases with output, $\delta\alpha^*/\delta x_1 = b^2c^2sp_1/(c^2sp_1 - b^2x_1)^2 > 0$. A larger sector might have increased political ability to persuade the regulator to set a higher α. The third equation in (3.4) becomes $\alpha\Sigma_i a_{i1}dw_i + \alpha' c_1 dx_1 = dp_1$ where c_1 is the original cost in the sector. The determinant of this model is $\Delta_f = c^2\alpha' c_1 s + \alpha b^2 > 0$ where $c = a_{12} + a_{22}$ and the w_{ij} results are:

$$\begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{pmatrix} = \begin{pmatrix} a_{22}b/\Delta_f & -a_{12}b/\Delta_f \\ (\alpha' c_1 cs - \alpha a_{21}b)/\Delta_f & (\alpha' c_1 cs + \alpha a_{11}b)/\Delta_f \end{pmatrix} = \underset{(a)}{\begin{matrix} + & - \\ - & + \end{matrix}} \quad \underset{(b)}{\begin{matrix} + & - \\ + & + \end{matrix}} \qquad (3.10)$$

The strong intensity price link holds for the monopoly price but is potentially relaxed in part for the price of the competitive product. An increase in p_2 would lower x_1 implying a decline in α, loss of monopoly power, and a potential increase in w_1. If the derivative α' approaches zero, the strong intensity price link remains intact. While ambiguity arises in the w_{12} term, cost would have to rise substantially to make it positive as in (3.10b). Specifically, α' would have to be larger than $\alpha a_{21}b/c_1cs$ to make w_{12} positive. Regardless, the intensity price link holds for the effects of the price of the noncompetitive product as well as the price of the competitive product on its intensive factor.

Consider an increase in p_2 holding p_1 constant. With $w_{22} > w_{12}$, percentage changes in prices and factor prices would be either $\hat{w}_2 > \hat{p}_2 > \hat{p}_1 = 0 > \hat{w}_1$ in (3.8a) or $\hat{w}_2 > \hat{p}_2 > \hat{w}_1 > \hat{p}_1 = 0$ in (3.8b). While w_1 may rise, the relative price of factor 1 would have to fall. The magnification effect would hold stated in terms of $\hat{c}_1$ instead of $\hat{p}_1$. The change in the real income of the owners of factor 1 becomes ambiguous while the real income of intensive factor 2 rises. If the owners of factor 1 consume little of good 1 their real income could rise if $\hat{w}_1 > 0$. Cost in sector 1 clearly rises with the increase in p_2 since $\delta c_1/\delta p_2 = a_{11}(\delta w_1/\delta p_2) + a_{21}(\delta w_2/\delta p_2) = \alpha' c_1 cds/\Delta_f > 0$ where $d \equiv a_{11} + a_{21}$. The output of sector 1 falls as does profit with a higher price in the competitive sector. With falling output, the monopoly loses monopoly power.

The crucial point for the present section is that competitive pricing is not necessary for the intensity price link. While some behavioral mechanisms for noncompetitive pricing certainly relax the intensity price link, competitive pricing is not necessary. In the present model, the intensity price link is consistent with monopoly.

9 Increasing Returns and the Intensity Price Link

Increasing returns with economies of scale external to the firm in the terminology of Marshall (1930) may affect the intensity price link. Firms hire inputs as though their output decision has no effect on factor prices even though total output of all firms affects technology in this industrial structure. Inoue (1981) extends the Stolper–Samuelson theorem to include variable returns. Increasing returns can lead to a convex production frontier as shown by Chipman (1965), Jones (1968), Mayer (1974), and Panagariya (1983). Helpman and Krugman (1986, chapter 3) show that factor price equalization holds for some distributions of endowments across countries with increasing returns. Chipman (1970) develops a model with parametric external economies and Thompson and Ford (1997) examine the corresponding production frontiers, contract curves, relative price lines, and intensity price link.

The foundation of increasing returns is a production function of the form $\xi_1 = \mathrm{f}(\upsilon; x_1)$ where ξ_1 is output of a typical firm in sector 1, υ is its input vector, x_1 is output of the sector, and $\delta\xi_1/\delta x_1 > 0$. In the literature on increasing returns, production functions are typically assumed to be separable in output, $\xi_1 = \mathrm{h}(x_1)\mathrm{f}(\upsilon)$ where $\mathrm{f}(\upsilon)$ exhibits constant returns and $\mathrm{h}' > 0$. Increasing returns occur, however, in a much wider class of production functions.

Variable returns can be generally specified with cost minimizing inputs a_{i1} functions of output as well as the vector $\mathbf{w}$ of factor prices, $a_{i1}(\mathbf{w}, x_1)$. A change in a_{i1}