INTERNATIONAL TRADE WITH THREE FACTORS, GOODS OR COUNTRIES

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Abstract: Three is the smallest number for illustrating some properties of the theory of production and trade. The number two is convenient but leads to incomplete intuition. The present paper synthesizes the literature on high dimensional trade theory and presents some new results using models with three factors, goods, or countries.

The theory of production and trade is two dimensional, based on exports and imports, traded goods and nontraded goods, capital and labor, home and foreign countries, and so on. Most theorems are proven under assumptions of two. While high dimensional models with many factors and many goods have been studied, generalizations from two dimensional trade theory have been scarce. The present paper focuses on the first step from two to many, namely three, where some concrete properties emerge. The higher dimensional literature is summarized in terms of three factors and three goods, and new results due to relabelling factors and goods are presented. Possible patterns of trade with three goods between three countries are derived.

1. THREE FACTORS

There is ample motivation for considering three factors. Classical economics is based on production with capital, labor, and land. Natural resources are in fact relevant for modelling the production and trade of many countries. Alternatively, much of the debate over free trade centers on the fate of unskilled versus skilled wages.

Neoclassical trade theory is built on production frontiers and offer curves. Production with two factors is used to derive the concave production frontier with two goods. Behind the two good production frontier there could, however, be three or more factors.

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Complementarity is reflected by a negative aggregate cross price substitution term, $s_{ik} = \Sigma_j x_j \partial a_{ij}/\partial w_k = \Sigma_j x_j \partial^2 c_j/\partial w_i \partial w_k < 0$ where $a_{ij} = \partial c_j/\partial w_i$ due to Shephard's lemma. A negative s_{13} implies $\partial^2 c_j/\partial w_1 \partial w_3 < 0$ for at least one good j. A higher w_3 must lower the marginal effect of w_1 on c_j.

Complementarity can be subtle. Consider the production function

$$x = v_1 + v_2 + v_3 - .5v_1^2 - .5v_2^2 - .3v_1v_2, \tag{2}$$

where v_i represents the input of factor i. Consider the unit isoquant $x = 1$, and restrict inputs so marginal products of factors 1 and 2 are positive. There is diminishing marginal productivity for factors 1 and 2, and $f_{33} = 0$. Cross partials are negative between factors 1 and 2: $f_{12} = f_{21} = -.3$. Suppose cost minimization occurs where $v_1 = v_2 = .4$ and $v_3 = .408$. The related symmetric matrix of Allen elasticities is

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} -25.1 & 7.43 & -4.18 \\ \cdot & -25.1 & 4.18 \\ \cdot & \cdot & -2.01 \end{bmatrix} \tag{3}$$

The unit isoquant is only slightly tilted and has the usual concave surface.

If one of the two goods in the 3×2 model is disaggregated, FPE would return. This reappearance of FPE may seem odd, but the $\partial w/\partial v$ results are nearly zero in the 3×2 model. In a quantitative sense, it matters little whether FPE holds. The implication is that factor endowment changes or differences have little (or no) influence on factor prices across freely trading, competitive, fully employed countries. If endowments change or differ, output changes or differences accomplish most of the adjustment to maintain full employment. Assuming no factor intensity reversals, global factor endowment differences are consistent with nearly equal factor prices.

Another lesson of the 3×2 model is that a factor intensity reversal (FIR) may occur with homothetic production. The intuition that FIRs are impossible is based on the 2×2 production box. In the 3×2 model, an FIR can occur without the contract curve crossing the diagonal. There are six different factor intensities corresponding to six parallelopipeds in the three dimensional production cube. With CES production, Wong (1990) shows factor intensity reversals cannot occur due to price changes, a result that applies to models with any number of factors and two goods. It remains an open issue, however, whether FIRs are possible with homothetic production. The observation of different factor intensities across countries has been taken to indicate the presence of FIRs or nonhomothetic production, but may only indicate the presence of three or more factors of production.

The intuition of the specific factors model is due partly to the assumption that each sector uses only two inputs. In the simplest version of the specific factors model, each of the two sectors has its own specific capital while labor is mobile. Its prominent intuition is that tariffs protect specific factors. In a model with two shared inputs, however, a tariff may lower the price of the specific factor. Neary (1978) makes the point that factor intensity reversals do not occur in the adjustment from the short run to the long run. Reversals may occur, however, if each sector employs three factors.

2. THREE GOODS

With two goods and two countries, each country will export one of the goods. There is one possible trade pattern in the sense that each country will export one good. In a model with three or more goods and fixed input proportions, Jones (1956) shows that trade depends on a ranking of factor abundance much like the model with two goods.

In a small open economy with more goods than factors, prices of all goods cannot be arbitrary. With as few as three goods, a higher price for one good can cause output of another good to rise. Goods are then complementary in production, demand and input-output considerations aside.

In a model with three goods and two countries, there are two trade patterns. The export matrix X for a given pair of countries A and B is

$$\begin{bmatrix} X_{1A} & X_{2A} & X_{3A} \\ X_{1B} & X_{2B} & X_{3B} \end{bmatrix}, \tag{4}$$

where X_{jk} is the excess supply of good j from country k. Rescale goods to unit prices. Balanced trade implies rows sum to zero, $\Sigma_j X_{jk} = 0$, $k = A, B$. Since the exports of one country equal imports of the other, the columns also sum to zero $\Sigma_k X_{jk} = 0$ for each good j. Label goods so $X_{1A} > 0$ and $X_{3B} > 0$. If $X_{2A} < 0$, switch labels on countries A and B and goods 1 and 3 to arrive at the possible trade patterns

$$\begin{array}{ccc} + + - & & + \ 0 \ - \\ - - + & & - \ 0 \ + \\ \text{(a)} & & \text{(b)} \end{array} \tag{5}$$

The middle good is not traded due to identical prices across countries in (5b), but the probability of this outcome is nearly zero.

In the 2×3 model there are less factors than goods, $r < n$. There is an unambiguous ranking of relative inputs,

$$a_{11}/a_{21} > a_{12}/a_{22} > a_{13}/a_{23}\,. \tag{6}$$

The production frontier has flat areas, similar to the 1×2 fixed proportions model. In a small open economy facing exogenous prices, specialization is expected at the "corners" of the production frontier. Possible sign patterns in the 2×3 model are those in (5), but the middle industry may shut down in either country.

The 3×3 model is a higher dimensional even model with the same number of factors and goods. Chipman (1965–6, 1988), Kemp and Wegge (1969), Inada (1971), Nikaido (1972), Uekawa, Kemp, and Wegge (1974), Jones and Scheinkman (1976), Egawa (1978), Takayama (1982), and Neary (1985) have generalized theorems from the 2×2 model. Minabe (1967), Chipman (1969), and Uekawa (1971) explicitly utilize the 3×3 model.

Factor price equalization generally occurs in the 3×3 model due to an invertible mapping between prices and factor prices. Only when isoquants are imbedded would this local univalence break down, creating the potential of global FIRs across countries.

Imbedding of isoquants, however, has almost zero probability. Local FPE is a general condition in the 3×3 model.

Price changes affect factor prices. Differentiate the competitive pricing condition $\Sigma_i a_{ij} w_i = p_j$, and use the cost minimizing envelope result $\Sigma_i w_i da_{ij} = 0$ to find

$$\begin{bmatrix} \theta_{11} & \theta_{21} & \theta_{31} \\ \theta_{12} & \theta_{22} & \theta_{32} \\ \theta_{13} & \theta_{23} & \theta_{33} \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{w}_3 \end{bmatrix} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix}, \tag{7}$$

where factor share $\theta_{ij} = a_{ij} w_i / p_j$, and ^ denotes percentage change. Effects of prices on factor prices are described by the inverse matrix θ^{-1}, $\theta^{-1}\hat{p} = \hat{w}$. Rows of θ' sum to one, as do rows of $\theta^{-1} : \theta^{-1}1 = \theta^{-1}(\theta 1) = (\theta^{-1}\theta)1 = I1 = 1$, where 1 is the 3×1 unit vector.

In the 2×2 model, the determinant of the factor share matrix is $|\theta| = \theta_{11}\theta_{22} - \theta_{12}\theta_{21}$. Label factors so $|\theta| > 0$. A tariff more than proportionately raises the price of "its" factor and lowers the price of the other factor, the magnification effect. The diagonal of θ^{-1} must have elements greater than 1 since θ_{11} and θ_{22} are both greater than $|\theta|$. The two elements off the diagonal of θ^{-1} must be negative, since the rows of θ^{-1} sum to one.

In the 3×3 model, θ is a Leontief matrix if θ^{-1} has a positive diagonal with $\hat{w}_i/\hat{p}_i > 0$ and negative elements off the diagonal, $\hat{w}_i/\hat{p}_k < 0$, $i \neq k$. A strong Stolper–Samuelson result occurs additionally if diagonal elements are greater than one, $\hat{w}_i/\hat{p}_i > 1$, and θ is then called a Minkowski matrix. Every Minkowski matrix is a Leontief matrix. A strong Stolper–Samuleson result corresponds closely to the intuition from the 2×2 model, a tariff raising the real price of "its" factor and lowering other factor prices. This strong result occurs if for every factor i, $\hat{w}_i/\hat{p}_j > 0$ for only one good j since the rows of θ^{-1} sum to one.

A weak Stolper–Samuelson result occurs if $\hat{w}_i/\hat{p}_i > 1$ but some element off the diagonal of θ^{-1} is nonnegative. A tariff then more than proportionately raises the price of its factor, but does not lower some other factor price.

Denote determinants of the minors of the elements of θ as m_{ik}. Inverting θ,

$$\theta^{-1} = \begin{bmatrix} m_{11} & -m_{21} & m_{31} \\ -m_{12} & m_{22} & -m_{32} \\ m_{13} & -m_{23} & m_{33} \end{bmatrix} \div |\theta| \tag{8}$$

Chipman (1969) uses the property that a determinant is unchanged if a column is added to another to simplify the determinant $|\theta|$. Adding the second and third columns to the first,

$$|\theta| = \begin{vmatrix} 1 & \theta_{21} & \theta_{31} \\ 1 & \theta_{22} & \theta_{32} \\ 1 & \theta_{23} & \theta_{33} \end{vmatrix} = m_{11} - m_{12} + m_{13}. \tag{9}$$

Similarly, $|\theta| = -m_{21} + m_{22} - m_{23} = m_{31} - m_{32} + m_{33}$.

If determinants of the principal minors m_{11}, m_{22}, and m_{33} are positive, θ is called a P matrix. If $\theta_{ii} > \theta_{ij}$ for $j \neq i$, the largest element of each column is in the diagonal and

θ is a P matrix. Relabelling factors switches the columns of θ and relabelling goods switches rows. Factors can be relabelled to obtain the property $|\theta| > 0$.

Chipman (1969) proves that if the share of factor i is greater in industry i than in any other industry, $\theta_{ii} - \theta_{ij} > 0$, $j \neq i$, the diagonal elements of θ^{-1} are greater than 1. The first element in the diagonal of θ^{-1} is $m_{11}/|\theta|$ which is greater than one iff $m_{11} > m_{11} - m_{12} + m_{13}$ or $m_{12} - m_{13} > 0$. The sign of $m_{12} - m_{13} = \theta_{21}(\theta_{33} - \theta_{32}) + \theta_{31}(\theta_{22} - \theta_{23})$ is positive under Chipman's assumption. Similarly, each element in the diagonal of θ^{-1} is greater than one under the Chipman assumption. Under the weak Stolper–Samuelson condition, every price more than proportionately affects some factor price. For every good j, there is a factor i such that $\hat{w}_i/\hat{p}_j > 1$. Every factor price is also more than proportionately affected by the price of some good. For every factor i, there is a good j such that $\hat{w}_i/\hat{p}_j > 1$.

Kemp and Wegge (1969) show there is a strong Stolper–Samuelson result in the 3×3 model if $\theta_{ii}\theta_{kj} - \theta_{ij}\theta_{ki} > 0$, for $k, j \neq i$. Their assumption directly implies that all elements off the diagonal in the adjoint of θ are negative and the diagonal elements are positive. The Kemp–Wegge assumption implies that θ is a P matrix and $|\theta| > 0$. Also, $m_{12} - m_{13} > 0$ and the diagonal elements of θ^{-1} are greater than 1.

Ethier (1974) shows that an increase in the price of a good lowers some factor price and more than proportionately raises another factor price. Every column of θ^{-1} has a negative element and an element greater than 1. Signs of elements in the columns of θ^{-1} are related. For instance, if m_{11} and m_{31} have the same sign, m_{21} must have the same sign. If $m_{11}, m_{31} > 0$, then $m_{21} > 0$ and the first column in (10) has one negative element. If $m_{11}, m_{31} < 0$, then $m_{21} < 0$ and the first column in (10) has two negative elements and one positive element. Otherwise, m_{11} and $_{31}$ have opposite signs.

Jones (1976) defines factor k as unimportant if $\Sigma_j \theta_{kj} < 1$. The sum of column k in matrix θ is then less than 1. For every unimportant factor k, there is a price p_m for which $w_k/p_m > 1$. This element can be assigned to the main diagonal by labelling.

The barycentric triangle technique of McKenzie (1955) and Leamer (1987) is used by Jones and Marjit (1991) and Jones (1992) to develop properties of factor intensity in the 3×3 model. Their analysis develops various degrees of factor abundance, using the reciprocity relationship $\partial w/\partial p = \partial x/\partial v$ to search for generalizations of the Stolper–Samuelson relationship.

Implications of relabelling factors or goods have not been explored. In the 3×3 model, there are six ways to number factors (123, 132, 213, 231, 312, 321) and similarly six ways to number goods. There are $6^2 = 36$ possible arrangements of elements for any θ matrix, but only six structurally different θ matrices. Switching labels on factors 1 and 2 and then goods 1 and 2, for instance, yields a θ matrix with the same elements in the main diagonal. The main diagonal elements from the minor of a moved element become its accompanying main diagonal elements.

Let F_{ijk} represent the ordering of factors and G_{ijk} represent the ordering of goods. For instance, $F_{132}G_{123}$ switches factors 2 and 3 but leaves goods in the same order. There are six groups of structurally similar θ matrices. Holding the ordering of goods constant at G_{123} and reordering factors leads to the following elements along the main diagonal of θ,

$$\begin{array}{llccc}
\text{(a)} & F_{123} & \theta_{11} & \theta_{22} & \theta_{33} \\
\text{(b)} & F_{132} & \theta_{11} & \theta_{23} & \theta_{32} \\
\text{(c)} & F_{213} & \theta_{12} & \theta_{21} & \theta_{33} \\
\text{(d)} & F_{231} & \theta_{13} & \theta_{21} & \theta_{32} \\
\text{(e)} & F_{312} & \theta_{12} & \theta_{23} & \theta_{31} \\
\text{(f)} & F_{321} & \theta_{13} & \theta_{22} & \theta_{31}
\end{array} . \tag{10}$$

There are six orderings for each of the cases in (10) that yield the same six elements along the diagonal of θ. For instance, (10a) results from the six orderings $F_{123}G_{123}$, $F_{132}G_{132}$, $F_{213}G_{213}$, $F_{231}G_{231}$, $F_{312}G_{312}$, and $F_{321}G_{321}$.

Properties of the stochastic θ matrix include unit sum rows and at least one negative sign in each row and column. These properties lead to the following four essential sign patterns of the θ^{-1} matrix,

$$\begin{array}{ccc}
+ & - & - \\
- & + & - \\
- & - & + \\
\multicolumn{3}{c}{\text{(a)}}
\end{array}
\quad
\begin{array}{ccc}
+ & - & - \\
- & + & - \\
+ & - & + \\
\multicolumn{3}{c}{\text{(b)}}
\end{array}
\quad
\begin{array}{ccc}
+ & + & - \\
- & + & - \\
+ & - & + \\
\multicolumn{3}{c}{\text{(c)}}
\end{array}
\quad
\begin{array}{ccc}
+ & + & - \\
- & + & + \\
+ & - & + \\
\multicolumn{3}{c}{\text{(d)}}
\end{array} . \tag{11}$$

In (11b) there is one good which "helps" two factors. The additional necessary condition for (11b) is that $m_{31} = \theta_{12}\theta_{23} - \theta_{13}\theta_{22} > 0$. In (11c) two goods each help two factors, and additionally $m_{12} < 0$. In (11d) each good helps two factors and additionally $m_{23} < 0$. Each of the four patterns in (11) can be arranged six ways as in (10). For instance, (11d) can appear in any of the following ways,

$$\begin{array}{ccc}
+ & + & - \\
- & + & + \\
+ & - & + \\
\multicolumn{3}{c}{(11d_1)}
\end{array}
\quad
\begin{array}{ccc}
+ & - & + \\
- & + & + \\
+ & + & - \\
\multicolumn{3}{c}{(11d_2)}
\end{array}
\quad
\begin{array}{ccc}
+ & + & - \\
+ & - & + \\
- & + & + \\
\multicolumn{3}{c}{(11d_3)}
\end{array}
\quad
\begin{array}{ccc}
+ & - & + \\
+ & + & - \\
- & + & + \\
\multicolumn{3}{c}{(11d_4)}
\end{array}
\quad
\begin{array}{ccc}
- & + & + \\
+ & - & + \\
+ & + & - \\
\multicolumn{3}{c}{(11d_5)}
\end{array}
\quad
\begin{array}{ccc}
- & + & + \\
+ & + & - \\
+ & - & + \\
\multicolumn{3}{c}{(11d_6)}
\end{array} . \tag{11$'$}$$

Any of ($11d_2$–d_6) can be relabelled to ($11d_1$). Relabelling factors is all that is necessary to find the six possible θ^{-1} arrangements. Relabelling factors alone will lead to the four possible outcomes for θ^{-1} in (11).

3. THREE COUNTRIES

Consider a world with three countries each producing two goods, with country A exporting good 1. One of the other countries, B or C, must then export good 2. Possible trade patterns are

$$\begin{bmatrix} X_{1A} & X_{2A} \\ X_{1B} & X_{2B} \\ X_{1C} & X_{2C} \end{bmatrix} = \begin{array}{cc} + & - \\ - & + \\ - & + \\ \multicolumn{2}{c}{\text{(a)}} \end{array} \quad \begin{array}{cc} + & - \\ 0 & 0 \\ - & + \\ \multicolumn{2}{c}{\text{(b)}} \end{array} , \tag{12}$$

analogous to (5). In (12a), countries B and C both export good 2 to country A. Countries and goods can be relabelled whenever two countries export one good to arrive at (12a). Excess demands from countries B and C are summed to match excess supply from A. Countries B and C do not trade with each other. Offer curves for B and C are summed along terms of trade vectors to meet A's offer curve. Contries B and C could be aggregated into a trade cohort.

In (12b), country B reverts to autarky under free trade with world prices equal to its autarky prices, a condition with almost zero probability. With m countries and n goods, $m > n$, autarky is possible for $m-n$ countries. Suppose world prices match autarky prices in country B. If the relative price of good 1 exogenously rises, country B enters a cohort with country A and exports good 1. If $m > n$, such export switching can occur for $m-n$ countries.

With three countries producing three goods, each country must export at least one good, and each good must be exported by at least one country. Without loss of generality, label countries so A exports good 1, B exports good 2, and C exports good 3. In the excess supply matrix

$$\begin{bmatrix} X_{1A} & X_{2A} & X_{3A} \\ X_{1B} & X_{2B} & X_{3B} \\ X_{1C} & X_{2C} & X_{3C} \end{bmatrix}, \tag{13}$$

the main diagonal is then positive. Balanced trade implies that each country must have an imported good, or each row in (13) must have at least one offsetting negative sign. Each good must have an importing country, or each column must have at least one negative sign.

Assume no two countries have the identical trade pattern, so there are no trade cohorts. Countries and goods can be labelled to reduce the possible trade patterns to

$$\begin{matrix} + & - & - \\ - & + & - \\ - & - & + \\ & \text{(a)} & \end{matrix} \qquad \begin{matrix} + & - & - \\ - & + & - \\ + & - & + \\ & \text{(b)} & \end{matrix} \qquad \begin{matrix} + & + & - \\ - & + & + \\ + & - & + \\ & \text{(c)} & \end{matrix} \qquad \begin{matrix} + & + & - \\ - & + & + \\ + & - & + \\ & \text{(d)} & \end{matrix} \tag{14}$$

In (14a), each country exports its good and imports the other two goods. In (14b), one country exports two goods. In (14c), two countries export two goods. In (14d), each country exports two goods. With the restrictions, there are six possible patterns for each of (14a–d), but all can be rearranged to put positive signs on the diagonal. The trade patterns in (14) are identical to the Stolper–Samuelson patterns in (11).

When there are more than two countries, trade can be balanced without having to balance trade with every trading partner. In (14a), for instance, country A exports $X_{1A}+X_{1C} = -X_{1B}$ to country B, and imports $X_{2B}+X_{2C} = -X_{2A}$ from country B. Countries A and B do not trade good 3. Balanced trade between the pair of countries A and B occurs only if $X_{1B} = X_{2A}$. Balanced trade with each trading partner has almost zero probability even though aggregate trade is balanced. This important principle is apparently disregarded in bilateral trade negotiations, partly a result of trade theory based on two countries.

4. CONCLUSION

Graham (1948), Pearce (1970), Chipman (1988), and others have consistently warned against projecting results from two dimensional trade theory. Working with three dimensional trade models makes this point clear, and the number three is small enough to catalogue possible outcomes. Issues of strategic trade policy, growth and trade, imperfect competition and trade, exchange rates, and the balance of payments can be examined in models with three factors, goods, or countries.

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