Instability of the radial flow over a rotating disk in a separated edgewise stream

Vrishank Raghav and Narayanan Komerath

Citation: Physics of Fluids (1994-present) **25**, 111701 (2013); doi: 10.1063/1.4833435
View online: http://dx.doi.org/10.1063/1.4833435
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof2/25/11?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF FLUIDS **25**, 111701 (2013)

Instability of the radial flow over a rotating disk in a separated edgewise stream

Vrishank Raghav[a] and Narayanan Komerath[b]
School of Aerospace Engineering, Georgia Institute of Technology, 270 Ferst Drive, Atlanta, Georgia 30332, USA

(Received 6 September 2013; accepted 11 November 2013; published online 22 November 2013)

We present velocity field measurements showing the instability of the radial flow over a rotating disk in a separated flow due to an edgewise freestream. A uniform edgewise stream significantly modifies the radial jet profile. Under separated flow conditions, co-rotating vortical structures form at the edge of the radial jet layer. The results establish that the discrete structures formed in the breakup of the radial flow layer on a rotor blade in retreating blade stall are also seen in the case of flow over a rotating disk. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4833435]

Engineers have long been puzzled by the fact that the airloads on a helicopter or wind turbine rotor blade appear to not show the expected effects of radial acceleration. Some years ago, our research team discovered that the radial jet layer immediately above the surface of a rotor blade in retreating blade stall broke up into discrete co-rotating vortical structures.[1,2] These structures attenuated the growth of the radial jet and hence demonstrated to have a first-order significance to the airloads on a rotating blade. Capturing such structures using computational fluid dynamics poses extreme demands on spatial and temporal resolution, as the structures must arise from some instability of the radial jet layer. A quasi-analytical way to predict the spacing, and thus the strength, of these structures would allow a breakthrough in such difficult problems as predicting the flow field features, and airloads in dynamic stall, which occurs on helicopters in forward flight and wind turbines in yaw. We hypothesized that fundamental insights to such predictions on a rotor blade could be developed from the flow over a rotating disk, which could in turn be linked to analytical solutions. An analytical solution is available for the steady axisymmetric three-dimensional boundary layer flow on an infinite rotating disk.[3,4] An analogous solution has been used in the past to help understand the flow over a rotating helicopter rotor blade at small angles of attack.[5] If structures similar to those observed on the rotor blade could be found over a rotating disk that would open the way to link the rotor blade phenomena to quasi-analytical solutions, and hence derive the fundamental relationships between parameters for the rotor blade problem. This would assist the development of computational prediction methods for helicopter rotors, among other applications.

Studies of dynamic stall using pitching and plunging airfoils in nominally two-dimensional conditions[6,7] cannot capture the effects of reactive centrifugal forces that are present on a rotating blade. These effects become especially significant and apparent on the retreating blade side, through the stall and reattachment phases. Recent investigations[1,8] demonstrated that the core of the dynamic stall vortex has a significant radial velocity component. Our team's work[1,2] determined that a strong radial jet developed close to the surface, but its growth was then attenuated as discrete co-rotating vortices structures broke off from the shear layer (see Fig. 1). The energy of these fluctuations was clearly originating from the radial jet near the surface, not from the free edge of the separated flow. This phenomenon was apparently triggered due to a shear layer instability at the edge of the radial jet layer. The spacing of these structures appeared to be fairly regular, but not tied to any evident periodicity of the flow. A fundamental understanding of these co-rotating structures would allow

[a]Electronic mail: vrishank@gatech.edu
[b]Electronic mail: komerath@gatech.edu

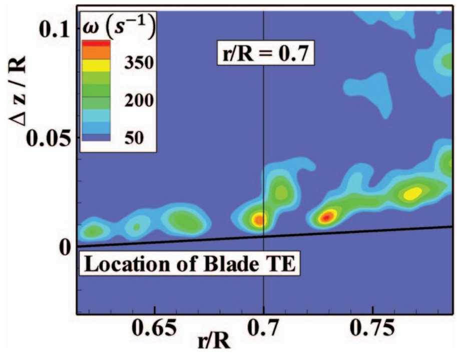
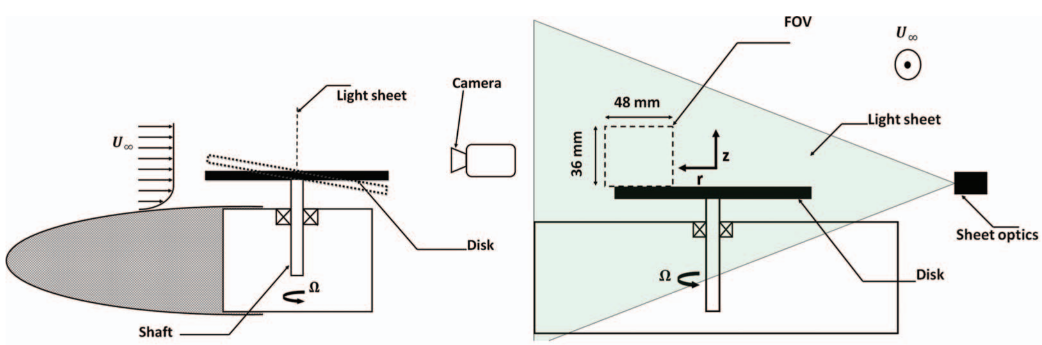


FIG. 1. Vorticity plot showing the co-rotating vortical structures on a rotating blade during dynamic stall conditions. Reprinted with permission from V. Raghav and N. Komerath, J. Am. Helicopter Soc. **58**, 022005 (2013). Copyright 2013, AHS International.

prediction of the limiting strength of the radial jet, and hence a better prediction of the airloads on the rotor blade through the critical regime of dynamic stall. As mentioned earlier, predicting the occurrence and properties of these structures from first principles through large-scale computational fluid dynamics is difficult, as they appear to stem from instabilities developing from very small spatial and temporal scales in the radial jet layer. Hence an approach to provide guidance on their characteristics was sought. This led to the investigation of an analogy with the rotating disk flow in a separated edgewise stream.

This letter accordingly reports on experiments conducted on a finite rotating disk. Particle image velocimetry (PIV) is used to investigate the radial flow over the rotating disk with and without a superimposed uniform stream to study the differences. First, the effect of a uniform stream on the radial velocity component on a rotating disk at zero angle of attack (α) is investigated. This is followed by experiments where a large separated flow region is induced on the rotating disk (by setting it at $\alpha = 8^{\circ}$) and the instability of the radial flow is studied. In summary, the primary objective is to demonstrate that the radial flow instability in the presence of a separated uniform stream is a fundamental behavior of the radial flow over a rotating surface.

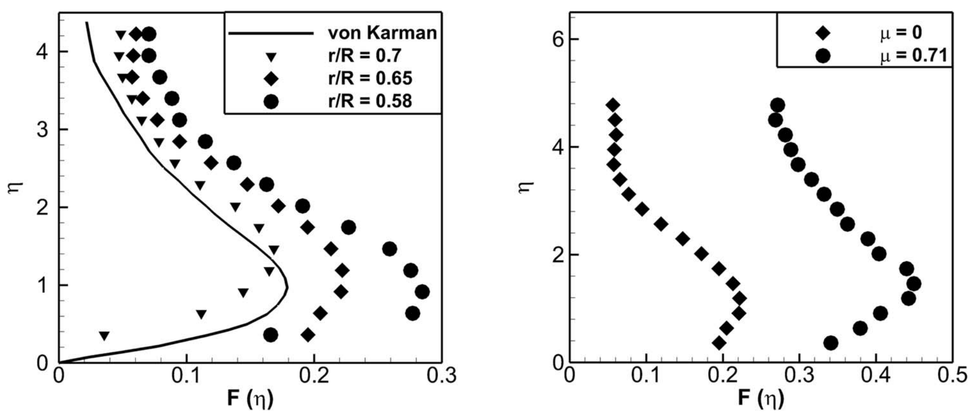
The experiments were conducted in the John Harper 2.13 m $\times$ 2.74 m low speed wind tunnel at the Georgia Institute of Technology. The closed circuit tunnel is powered by a three-phase 600 hp induction motor controlled by a variable frequency drive. The error in fan rotation speed is maintained below 0.1%. The turbulence intensity in the wind tunnel is 0.06% measured at a freestream velocity of 110 ft/s (33.5 m/s). The experimental setup sketched in Fig. 2 consists of a disk of diameter 154 mm spinning at an angular velocity Ω (rad/s) mounted on a stand inside the wind tunnel. The



(a) Side view with the dotted rotating disk indicating angle of attack setting

(b) Downstream view showing light sheet and field of view

FIG. 2. Illustration of the experimental setup (not to scale).



(a) μ=0

(b) Effect of uniform stream at r/R = 0.65

FIG. 3. PIV measurement of the mean radial velocity profiles at Re = 175.8 and $\alpha = 0^{\circ}$.

smooth elliptical surface fixed upstream served to provide a smooth freestream along the edge of the rotating disk. The shaft rotation rate (angular velocity) is measured using a digital encoder with an accuracy of 0.36°. The measured disk angular velocity was stabilized to within 0.12% of the angular velocity. Vibrations and out of plane motion of the rotating disk were confirmed to be insignificant by examination of images taken very close to the disk surface.

PIV was used to measure the velocity fields above the rotating disk, illuminated by a double-cavity Nd:YAG laser with a pulse energy of 200 mJ. A laser arm (a covered beam path) with sheet optics illuminated the measurement plane with a light sheet of 2 mm thickness. The flow was continuously seeded with 10 μm droplets from a fog generator. Since the mean flow direction was normal to the PIV image plane, the pulse separation (Δt) had to be short enough to keep out-of-plane movement below 1/3 of the sheet thickness.[9] At each measurement plane and flow condition, 100 image pairs were acquired using a PRO-X 2M camera with 1600 × 1200 pixel resolution and a pixel size of 7.4 × 7.4 μm^2. The lens system had a 60 mm focal length and an aperture of f/2.8. The disk was coated with black paint to improve signal to noise ratio, and the laser reflections at the disk surface were masked during the velocity vector computations. The LaVision DaVis 7.2 software was used to process the PIV data. Velocities were calculated from the spatial cross-correlation of the image-pairs. A 64 × 64 pixels interrogation window with 50% overlap and a second interrogation pass with a reduced window size (32 × 32 pixels) were used to yield a vector spatial resolution of 0.12 mm for boundary layer measurements and 0.48 mm for the "macro" flow field measurements. A median filter and light vector smoothing (3 × 3) were applied in post-processing for better visual presentation. The field of view (FOV) when studying the effect of the freestream on the radial boundary over the disk was 12 mm×9 mm.

In the following, the freestream velocity is non-dimensionalized using the tip speed of the disk $R\Omega$ (where R is the radius) and given by advance ratio $\mu = U_\infty/\Omega R$. Thus, a rotating disk in quiescent fluid would be indicated by $\mu = 0$ and a stationary disk in a freestream would be indicated by $\mu = \infty$. The flow field variables are non-dimensionalized using the displacement thickness $\delta = \sqrt{\nu/\Omega}$ (where ν is the kinematic viscosity of the fluid) and the local tangential velocity of the disk $r\Omega$ (where r is the local radius). The dimensionless height above the surface, time averaged radial velocity component, and Reynolds number are written as $\eta = z\sqrt{\frac{\Omega}{\nu}}$, $F(\eta) = \frac{u_r}{\Omega r}$, and $Re = R\sqrt{\frac{\Omega}{\nu}}$ respectively.

The von Kármán similarity solution tabulated by Cochran[4] gives the steady radial velocity $F(\eta)$ over a rotating **smooth infinite disk** in quiescent fluid. Figure 3(a) compares the measured mean radial velocity on the 154 mm rotating disk in quiescent fluid with the above. The results reveal the effects of roughness (due to spray painting the disk[10]) and of using a finite disk. The near-surface radial jet reaches a substantially higher peak value than that for the smooth infinite disk. This is

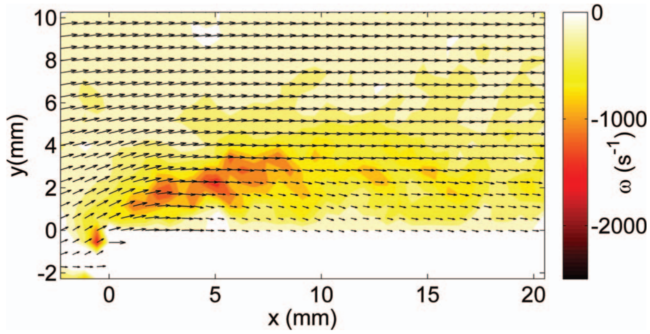


FIG. 4. Evidence of separation over the rotating disk at Re = 178, $\mu = 0.69$, and $\alpha = 8^\circ$ as viewed from the side.

expected since the roughness serves to effectively increase the momentum transfer across the flow near the surface. The roughness was measured using a stylus-based instrument (Profilometer), and found to be in the range to yield such an increase. The effect of a uniform stream on the radial velocity profile at Re = 175.8 is illustrated in Fig. 3(b). The addition of a freestream increases the total radial velocity component by a factor of two at r/R = 0.65. The *effective* radial velocity component (obtained by correcting for the radial velocity far away from the disk such that it approaches zero) increases by 87%. When a freestream is superimposed at zero angle of attack on a rotating disk, the presence of the strong radial flow causes the incoming flow to separate at the leading edge (as seen in Ref. 11). Furthermore, the influence of the freestream limited the growth of the peak radial velocity to half of the expected increase based on observations of the peak radial velocity without the freestream.

The above results showed that the addition of a freestream has a significant influence on the *effective* radial velocity component on the rotating disk. Next, the effect of a freestream on the "macro" flow field of the rotating disk was studied using a FOV of 48 mm × 36 mm. The angular velocity of the disk (expressed as advance ratio μ) was varied with the freestream held constant. An angle of attack of $\alpha = 8^\circ$ was sufficient to induce a separated flow on the upper surface of the rotating disk. The separation was verified using PIV as observed in Fig. 4, which is a view from the side of the rotating disk with the uniform stream from the left to the right (refer to Fig. 2(a) for clarification on the view). In the figure, (0,0) is the leading edge of the rotating disk.

Baseline experiments with a uniform stream at an angle of attack were conducted with no rotation of the disk. The separated flow had a small velocity component towards the disk surface, expected due to the separated flow conditions. With the disk rotating, the radial velocity component was observed to become unstable, revealing co-rotating vortical structures located above the radial boundary layer. Note that this is instability of the layer above the radial jet, not the thin boundary layer between the radial jet and the solid surface. Figures 5 and 6 illustrate a typical set of co-rotating vortices observed in instantaneous velocity fields for all flow conditions tested in this work at $\alpha = 8^\circ$. To improve the spatial resolution, the PIV data in Fig. 5 were processed with a 75% overlap in contrast to the regular 50% overlap in Fig. 6. There are spurious velocity vectors close to the disk

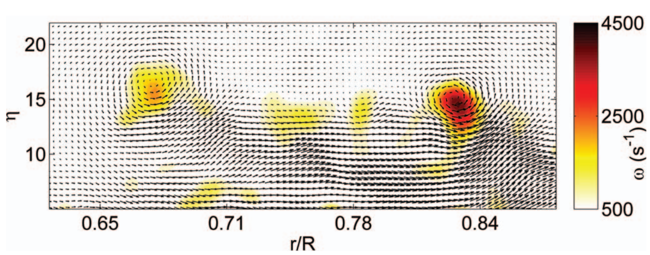


FIG. 5. PIV velocity vector field illustrating a typical set of co-rotating vortices above the rotating disk at $\alpha = 8^\circ$ for Re = 178 and $\mu = 0.69$.

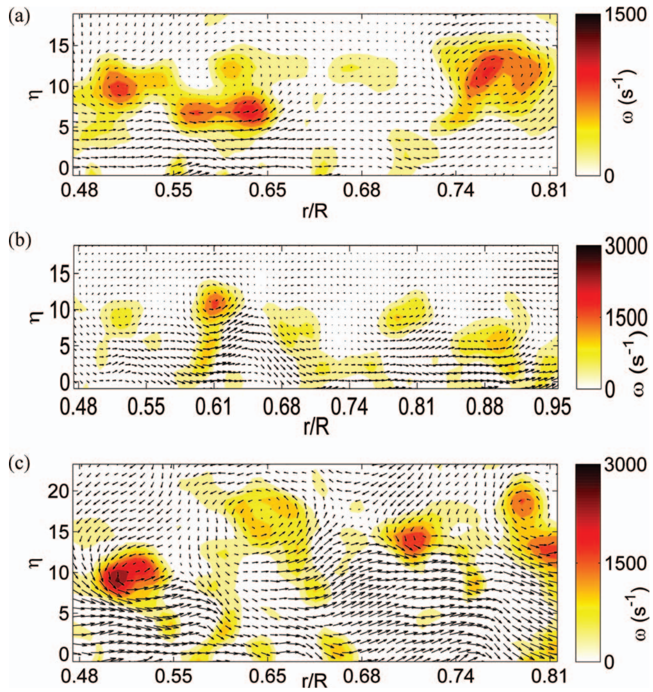


FIG. 6. PIV velocity vector field illustrating a typical set of co-rotating vortices above the rotating disk at $\alpha = 8^\circ$ for (a) Re = 126 and $\mu = 1.39$, (b) Re = 154 and $\mu = 0.92$, and (c) Re = 199 and $\mu = 0.55$.

surface ($\eta \sim 0$) due to the laser reflections. The vortical structures were observed only during the rotation of the disk and not otherwise, firm evidence that the instability is due to the radial flow itself and not an artifact of separation. Furthermore, these co-rotating vortices were not observed on the rotating disk in a quiescent medium at the same angular velocities.

Previous work on instability of the boundary layer flow on a rotating disk reveals that the layer is absolutely unstable for $Re \sim 507$[12] and is convectively unstable to non-stationary modes for $Re \sim 50$.[13] A reviewer has pointed out that the co-rotating vortices observed here could as well be the non-stationary modes due to the convective instability of the boundary layer with a superimposed uniform stream. Against this plausible hypothesis, it should be noted that the rotating disk is operated at an angle of attack under separated flow conditions. The vortical structures observed here are *not* consistent in occurrence with respect to the radial location, spacing/frequency (between each vortical structure), and phase velocity. Due to the nature of these vortical structures, all inferences are drawn from the conditional statistics of the PIV velocity fields. That is, only velocity fields where these structures appear are considered in the statistical analysis.

The conditionally averaged height of the vortical structures above the surface of the blade was determined for various angular velocities of the disk. The height above the disk surface at which these discrete structures were observed increases as the angular velocity increases (summarized in Table I). Although at first this is surprising, the analysis of the radial velocity profile at each angular velocity helped reconcile the observations. The time averaged radial velocity profiles for

TABLE I. Summary of the mean (η_m) and standard deviation (η_s) of the height above the disk surface of the co-rotating vortical structures.

Flow condition	η_m	η_s
Re = 126, $\mu = 1.39$	6.88	2.82
Re = 154, $\mu = 0.92$	12.05	2.97
Re = 178, $\mu = 0.69$	16.17	4.21
Re = 199, $\mu = 0.55$	23.21	6.85

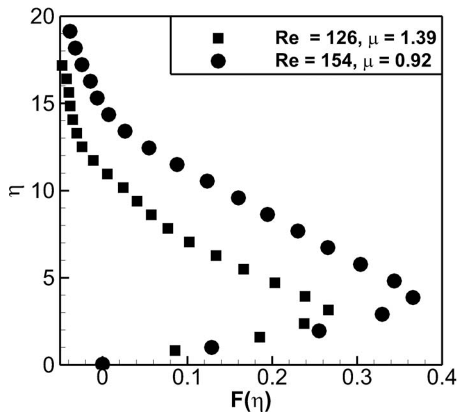


FIG. 7. Mean radial velocity profiles on the rotating disk at $\alpha = 8^{\circ}$ illustrating the increase in height of the inflection point at r/R = 0.59.

a rotating disk at $\alpha = 8^{\circ}$ are illustrated in Fig. 7 for r/R = 0.59. Due to flow separation, the flow adjacent to the disk surface moves at relatively low speed and hence, the centrifugal stresses at the surface are transported to a greater height above the surface, as clearly evident in Fig. 7. The critical observation here is that the inflection point of the time averaged radial velocity profile moves away from the surface of the disk with increasing angular velocity. This observation concurs with the average location of the co-rotating vortical structures, indicating that the vortical structures are due to an instability of the radial flow itself. The instability appears to be some form of the classical Kelvin-Helmholtz instability of a shear layer with an inflection point.

Having established the cause and source of the instability, the co-rotating vortical structures were further characterized. The co-rotating vortical structures are defined by strength (vorticity), frequency, and phase velocity. The angular velocity appears to have a direct effect on the vorticity entrained in the structures, with the vorticity magnitude increasing with rotation rate. This again supports the assertion that these vortical structures are an artifact of rotation and the associated radial flow. The vorticity entrained in these structures is expected to depend on the independent variables angular velocity Ω, freestream velocity U_∞, kinematic viscosity ν, and radius of the disk R. These variables can be grouped to form a non-dimensional vorticity given by $\omega^\star = \omega/\Omega$ which would be expected to scale with the parameters: $\kappa = \frac{{U_\infty}^2}{\nu\Omega}$ and $\mu = \frac{U_\infty}{R\Omega}$ with $\omega^\star = f(\kappa, \mu)$. The dimensional and non-dimensional vorticity entrained in these structures are summarized in Table II.

The radial spacing Δ of the co-rotating structures determined by conditional averaging was found to be 9 mm to 11 mm for the different experimental conditions. With freestream held constant, an increase in angular velocity of the disk resulted in decreased spacing of the structures (i.e., increased spatial frequency). The spacing is non-dimensionalized as $\Delta^\star = \Delta\Omega/U_\infty$ with $\Delta^\star = f(\kappa, \mu)$, where κ, μ are non-dimensional parameters defined earlier. Initial analysis of the data to determine the phase velocity of the structures suggests that in some instances the structures are convecting at the "local" radial velocity, which is a function of both r and η. A rigorous analysis would require data with higher temporal resolution.

TABLE II. Summary of the mean (subscript *m*) and standard deviation (subscript *s*) of vorticity entrained in the co-rotating vortical structures.

Flow condition	ω_m (s^{-1})	ω_s (s^{-1})	${\omega^\star}_m$	${\omega^\star}_s$
$\kappa = 30\,422$, $\mu = 1.39$	1055.2	379.90	25.19	9.06
$\kappa = 20\,281$, $\mu = 0.92$	1341.5	358.15	21.35	5.70
$\kappa = 15\,211$, $\mu = 0.69$	1508.2	370.98	18.01	4.42
$\kappa = 12\,169$, $\mu = 0.55$	1698.4	545.45	16.22	5.21

From the above observations and discussion, it appears that the radial flow instability leading to the formation of co-rotating vortices is a fundamental behavior of the radial component of the velocity with a uniform edgewise stream. This allows for an analogy between the flow features on the rotating disk and the rotating blade in similar flow conditions defined by non-dimensional parameters κ and μ. It opens a path to systematically study the radial flow instability without having to perform large scale rotor blade experiments or computational analyses on rotating blades. The analytical solution can in principle be modified to account for the uniform stream to check for the instability and then incorporated into the radial flow model on rotating blades.[5] Various hypotheses still remain to be explored, for instance those suggested by the reviewers, to seek a closed-form analytical prediction of the quantitative spatial frequency and strength of these structures, as related to relevant operating parameters. This in turn would lead to models that predict the limiting effect of radial flow on the aerodynamic loads experienced by rotating blades, especially through the dynamic stall regime. Hence this letter establishes a promising avenue to make progress on the dynamic stall problem and related aeromechanics of rotors.

In conclusion, we have shown that the radial flow on a rotating disk under separated flow conditions becomes unstable with the addition of a uniform stream. The instability leads to the formation and evolution of co-rotating vortices at the upper edge of the radial jet. The average height above the surface where the co-rotating structures form increases with angular velocity. This concurs with the increase in the height of the inflection point of the radial velocity profile above the surface. This clearly indicates that the source of the instability is the radial jet layer above the disk surface, and that the instability is *not* an artifact of separation of the freestream over the rotating disk. Vorticity entrained in these co-rotating structures has a direct correlation to the rotation rate, with the vorticity magnitude increasing with the angular velocity of the disk. The spacing of these structures was determined to be inversely proportional to the angular velocity of the rotating disk. The co-rotating vortices appear to convect with a finite phase velocity; however, a definite quantitative conclusion on the convection velocity requires time resolved data.

This work was funded by the Army Research Office (ARO), Grant No. W911NF1010398. The authors acknowledge the assistance of Alex Forbes with the experimental setup and measurements, and the assistance of Roger Lascorz with the post processing of data. We are also grateful to the anonymous reviewers for their valuable feedback and hypotheses on this work.

[1] J. DiOttavio, K. Watson, J. Cormey, N. Komerath, and S. Kondor, "Discrete structures in the radial flow over a rotor blade in dynamic stall," in *Proceedings of the 26th Applied Aerodynamics Conference, Honolulu, Hawaii, USA* (AIAA, 2008).
[2] V. Raghav and N. Komerath, "An exploration of radial flow on a rotating blade in retreating blade stall," J. Am. Helicopter Soc. **58**, 1–10 (2013).
[3] V. Kármán, "Über laminare und turbulente reibung," Z. Angew. Math. Mech. **1**, 233–252 (1921).
[4] W. Cochran, "The flow due to a rotating disc," in *Mathematical Proceedings of the Cambridge Philosophical Society*, Vol. 30 (Cambridge University Press, Cambridge, 1934), pp. 365–375.
[5] W. McCroskey and P. Yaggy, "Laminar boundary layers on helicopter rotors in forward flight," AIAA J. **6**, 1919–1926 (1968).
[6] L. W. Carr, "Progress in analysis and prediction of dynamic stall," J. Aircr. **25**, 6–17 (1988).
[7] K. Mulleners and M. Raffel, "The onset of dynamic stall revisited," Exp. Fluids **52**, 779–793 (2012).
[8] K. Mulleners, K. Kindler, and M. Raffel, "Dynamic stall on a fully equipped helicopter model," Aerosp. Sci. Technol. **19**, 72–76 (2012).
[9] M. Raffel, C. E. Willert, and J. Kompenhans, *Particle Image Velocimetry: A Practical Guide* (Springer-Verlag, Berlin, 1998).
[10] F. Zoueshtiagh, R. Ali, A. J. Colley, P. J. Thomas, and P. W. Carpenter, "Laminar-turbulent boundary-layer transition over a rough rotating disk," Phys. Fluids **15**, 2441 (2003).
[11] J. R. Potts and W. J. Crowther, "The flow over a rotating disc-wing," in *Aerodynamics Research Conference Proc., London, UK* (Royal Aeronautical Society, 2000).
[12] R. J. Lingwood, "An experimental study of absolute instability of the rotating-disk boundary-layer flow," J. Fluid Mech. **314**, 373–405 (1996).
[13] M. R. Malik, S. P. Wilkinson, and S. A. Orszag, "Instability and transition in rotating disk flow," AIAA J. **19**, 1131–1138 (1981).