Uniquely bipancyclic graphs on more than 30 vertices*

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Abstract

A bipartite graph on n vertices, n even, is called uniquely bipancyclic (UBPC) if it contains precisely one cycle of length $2m$ for every $2 \leq m \leq n/2$. In this note, using computer programs, we show that if $32 \leq n \leq 56$, and $n \neq 44$, then there are no UBPC graphs of order n. We also present the six non-isomorphic UBPC graphs of order 44. This improves the recent results on UBPC graphs of order at most 30.

Keywords: pancyclic graphs, uniquely pancyclic graphs, uniquely

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bipancyclic graphs
MSC 2000: 05C69

1 Introduction

All graphs in this paper are finite, simple and undirected. We use [5] for the terminology not defined here. A *pancyclic* graph of order n is a graph that contains cycles of every length from 3 to n (see [1]). A pancyclic graph with exactly one cycle of every possible length is called *uniquely pancyclic* [3, 2]. A bipartite graph on n vertices, n even, is called a *uniquely bipancyclic* (UBPC) if it contains precisely one cycle of length $2m$ for every $2 \leq m \leq n/2$. The concept of uniquely bipancyclic graphs was recently introduced in [4]. The author of [4] finds all uniquely bipancyclic graphs on at most 30 vertices. Namely, up to isomorphism, there is only one uniquely bipancyclic graph of order 4 and only one of order 8. There are precisely four uniquely bipancyclic graphs of order 14 and six of order 26.

In this note, using computer programs, we show that if $32 \leq n \leq 56$, and $n \neq 44$, then there are no UBPC graphs of order n. We also present the six non-isomorphic UBPC graphs of order 44.

2 Search for uniquely bipancyclic graphs

A UBPC graph can be drawn as a Hamiltonian cycle with some added edges which we will refer to as chords. In [4] all UBPC graphs with three or fewer chords were classified. In what follows we classify all UBPC graphs with four or five chords.

Appendix A displays 17 layouts with four chords and Appendix B displays 75 layouts with 5 chords. Based on the number of pairs of chords that intersect, the 17 layouts can be split into 7 groups and the 75 layouts into 11 groups. For each group we draw all possible layouts of chords that give us the desired number of intersections. For each layout, we label each arc with a variable, where an arc is a path on the Hamiltonian cycle between adjacent chord endpoints. We then use these variables to describe the length of each cycle with an equation. Once we have these equations, we run a program that checks whether any combination of variable values results in a UBPC graph.

Note that a layout can simply be obtained from another layout by fixing some arc lengths at zero. Fixing an arc length at zero may reduce the number of cycles (and hence reduces the order) of a layout. Appendices A and B do not contain layouts with zero arc lengths. Any layout with zero arc lengths can be obtained from a layout in the appendices by fixing some arc lengths at zero. In addition, in a given layout we set the arc lengths at zero only if this does not reduce the number of chords or the number of intersections.

Example: Figure 1 displays a layout for possible UBPC graphs of order 32. Note that there are precisely 15 different cycles in this layout. In this figure, a_j indicates a path of length a_j on the Hamilton cycle for $1 \leq j \leq 8$. Clearly, we must have $\sum_{j=1}^{8} a_j = 32$. There are precisely nine other layouts, up to isomorphism, for order 32 which can be obtained from the layout in Figure 1 by fixing some arc lengths at zero. Let ℓ_i for $1 \leq i \leq 15$ be the lengths of the 15 cycles in the layout. A simple computer program shows that $\{\ell_i \mid 1 \leq i \leq 15\} \neq \{4, 6, 8, \ldots, 32\}$ for any arc length $0 \leq a_j \leq 32$, where $1 \leq j \leq 8$. Hence, there is no UBPC graph of order 32.

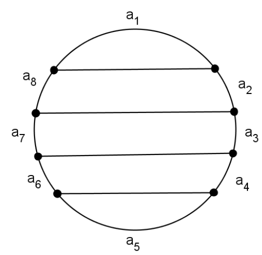

Figure 1: A layout for possible UBPC graphs on 32 vertices

We examined every layout given in Appendices A and B and found that:

Main Theorem

1. There is no uniquely bipancyclic graph of order n, where $32 \leq n \leq 56$ and $n \neq 44$.
2. There are precisely six non-isomorphic uniquely bipancyclic graphs of order 44. These graphs are displayed in Figure 2.

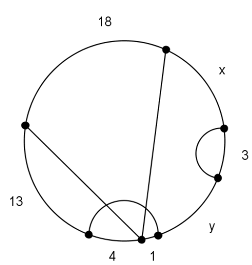

Figure 2: The six non-isomorphic uniquely bipancyclic graphs of order 44, where $x + y = 5$ and $0 \leq x \leq 5$.

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References

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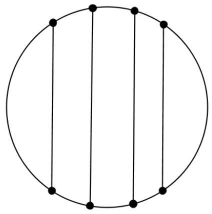
[2] K. Markstrom, *A note on uniquely pancyclic graphs*, Australasian Journal of Combinatorics **44** (2009), 105-110.

[3] Y. Shi, *Some theorems of uniquely pancyclic graphs*, Discrete Mathematics **59** (1986), 167-180.

[4] W. Wallis, *Uniquely bipancyclic graphs*, Journal of Combinatorial Mathematics and Combinatorial Computing (to appear).

[5] D.B. West, *Introduction to Graph Theory*, Prentice-Hall, Inc, 2000.

Appendix A: Layouts with four chords



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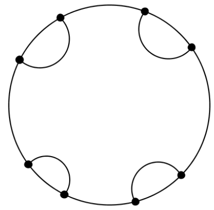

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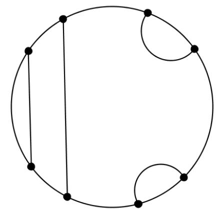

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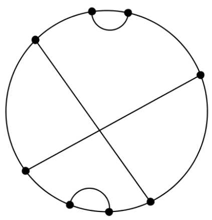

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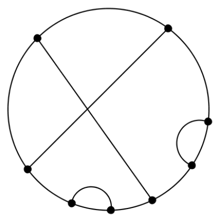

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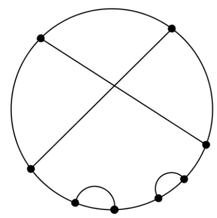

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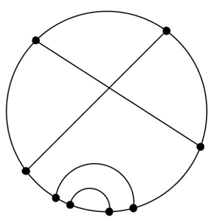

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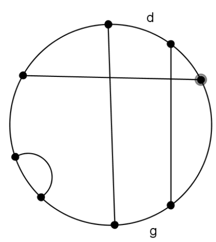
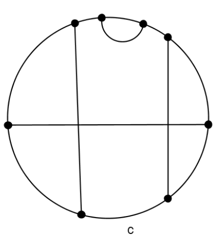
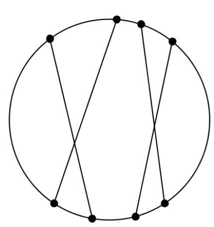


	g=0	d=0	otherwise
order	42	44	46



	c=0	otherwise
order	44	48



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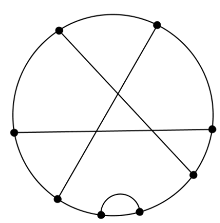

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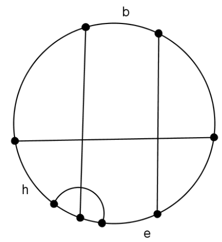
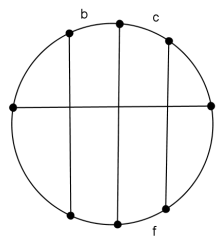
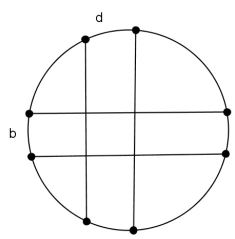


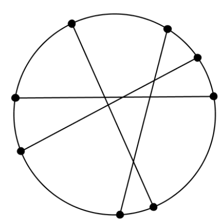
	h=0	e=0	h=e=0	b=0	b=h=0
order	50	52	48	50	48
	b=e=0	b=h=e=0	otherwise		
order	48	46	54		



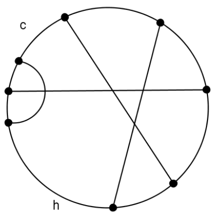
	b=0	b= f=0	b=c=0	otherwise
order	50	48	44	54



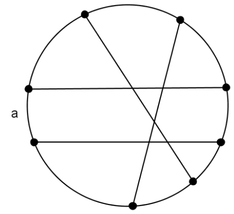
	b=0	b=d=0	otherwise
order	52	48	58



order=60

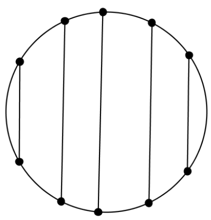


	c=0	c=h=0	otherwise
order	54	50	58



	a=0	otherwise
order	54	60

Appendix B: Layouts with five chords



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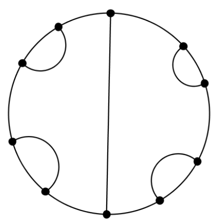
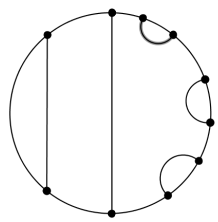
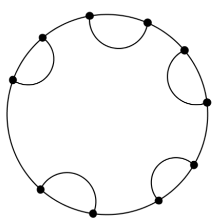


order=58



order=62



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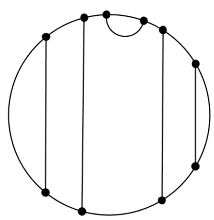

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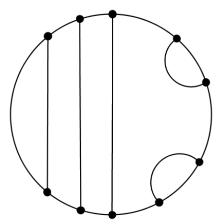

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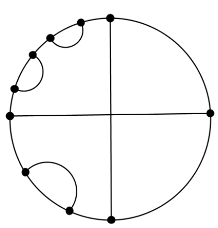

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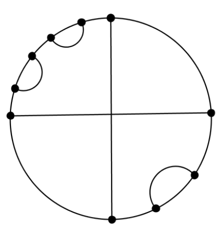

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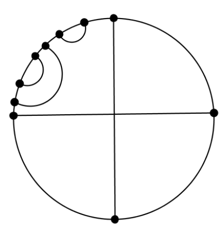

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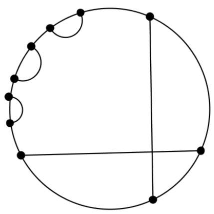

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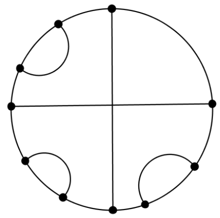

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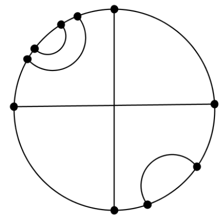

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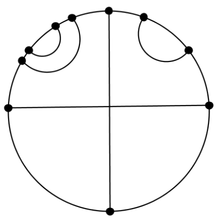

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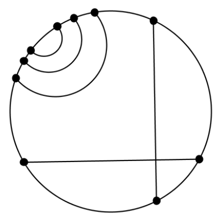

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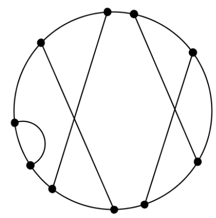

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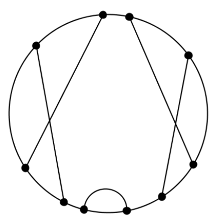

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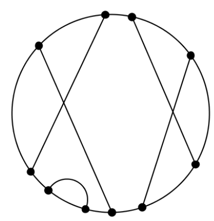

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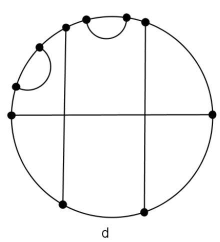
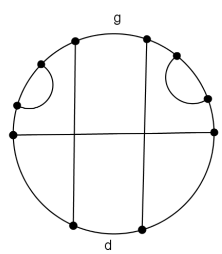


	d=0	otherwise
order	68	72



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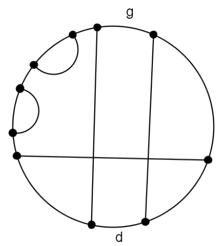
order	66	60	68



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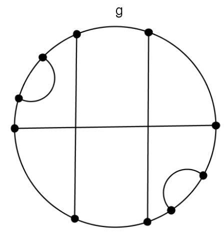
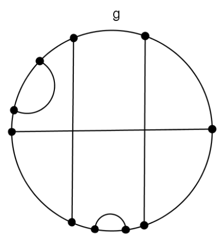
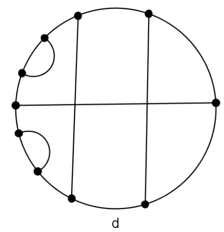
order	74	68	76



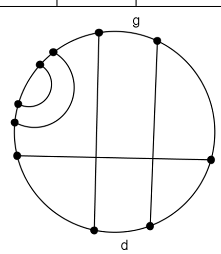
	g=0	otherwise
order	64	68



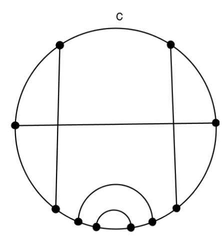
	g=0	otherwise
order	64	72



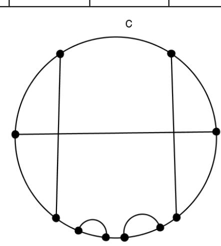
	d=0	otherwise
order	64	68



	d=0	g=0	otherwise
order	62	58	64



	c=0	otherwise
order	62	68



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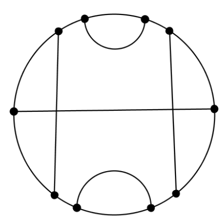

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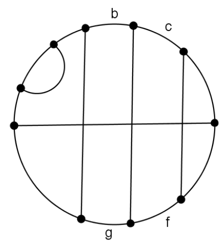
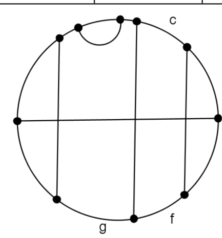


	b=0	c=0	g=0	f=0	b=f=0
order	74	76	74	70	72
	c=g=0	b=c=0	g=f=0	otherwise	
order	72	62	68	80	



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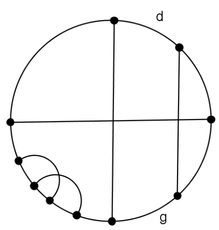
order	76	76	80	72	68	84



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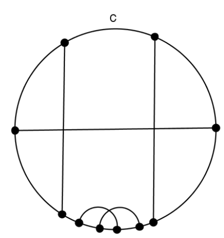
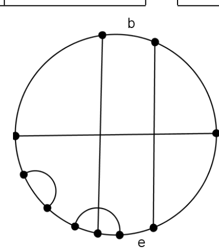
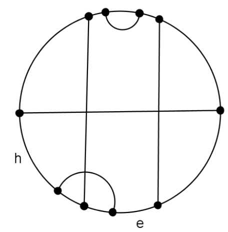
order	76	70	78



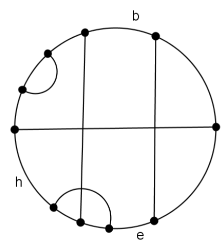
	c=0	otherwise
order	76	84



	b=0	e=0	b=e=0	otherwise
order	80	80	76	84



	e=0	h=0	e=h=0	otherwise
order	80	80	76	84



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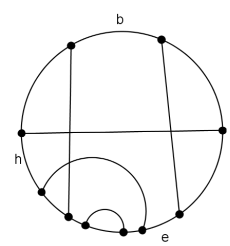
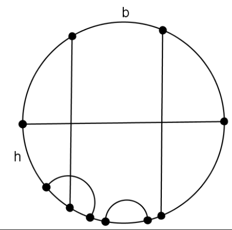
order	72	78	72	70	68
	e=h=0	b=e=h=0	otherwise		
order	70	66	80		



	b=0	e=0	h=0	b=e=0	b=h=0
order	74	76	76	70	72
	e=h=0	b=e=h=0	otherwise		
order	72	68	80		



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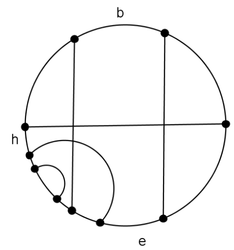
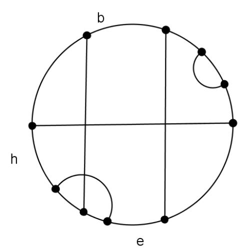
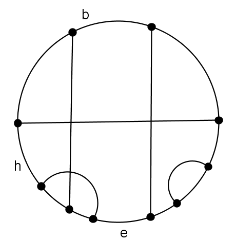
order	80	80	76	88



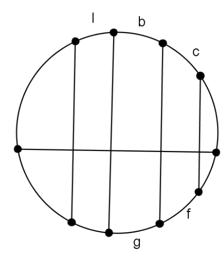
	b=0	e=0	h=0	b=e=0	b=h=0
order	74	78	72	72	70
	e=h=0	b=e=h=0	otherwise		
order	70	68	80		



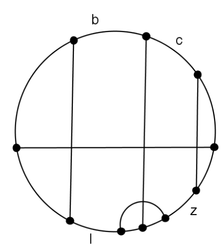
	b=0	e=0	h=0	b=e=0	b=h=0
order	72	78	74	70	70
	e=h=0	b=e=h=0	otherwise		
order	72	68	80		



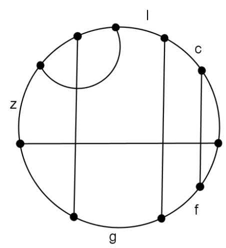
	b=0	e=0	h=0	b=e=0	b=h=0
order	76	76	74	72	72
	e=h=0	b=e=h=0	otherwise		
order	70	68	80		



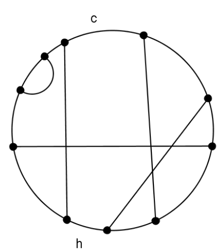
	ℓ=0	b=0	$\ell = b = 0$	$\ell = c = 0$	ℓ=g=0
order	88	88	76	82	84
	ℓ=f=0	ℓ=g=f=0	ℓ=b=c=0	ℓ=c=g=0	otherwise
order	80	74	64	80	96

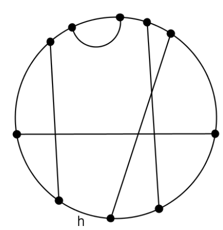


	b=0	ℓ=0	b=c=0	b=ℓ=0	c=ℓ=0
order	88	92	76	84	86
	b=c=ℓ=0	b=ℓ=z=0	otherwise		
order	74	82	96		



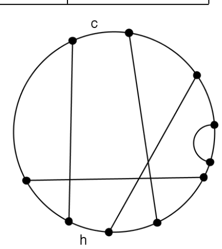
	c=0	f=0	g=0	ℓ=0
order	86	88	86	90
	z=0	c=g=0	c=ℓ=0	c=z=0
order	86	82	80	80
	f=g=0	f=ℓ=0	f=z=0	g=ℓ=0
order	76	86	80	82
	g=z=0	ℓ=z=0	c=g=z=0	c=ℓ=g=0
order	82	82	78	76
	c=ℓ=z=0	f=g=ℓ=0	f=g=z=0	f=ℓ=z=0
order	74	74	74	78
	g=ℓ=z=0	c=ℓ=g=z=0	f=g=ℓ=z=0	otherwise
order	78	72	72	94



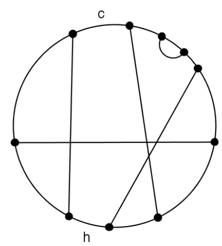


	c=0	h=0	c=h=0	otherwise
order	78	82	74	86

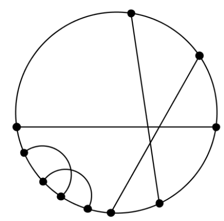
	h=0	otherwise
order	84	92



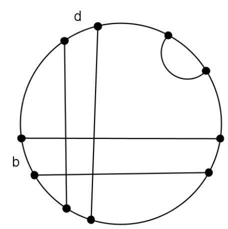
	c=0	h=0	c=h=0	otherwise
order	82	82	76	88



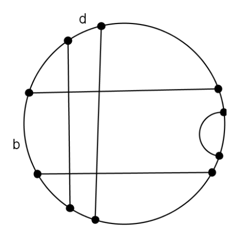
	c=0	h=0	c=h=0	otherwise
order	80	82	74	88



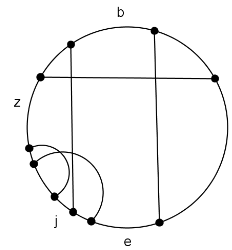
order=86



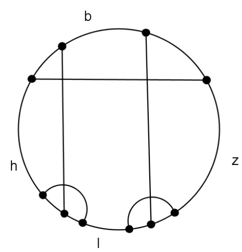
	b=0	d=0	b=d=0	otherwise
order	80	76	72	88



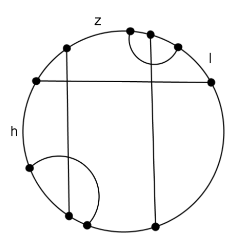
	b=0	d=0	b=d=0	otherwise
order	78	80	72	88



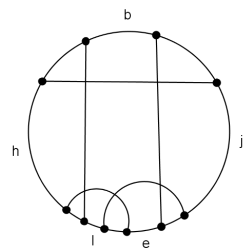
	b=0	e=0	j=0	z=0	b=e=0
order	88	90	88	90	84
	b=j=0	b=z=0	e=j=0	e=z=0	j=z=0
order	82	86	86	86	84
	b=e=j=0	b=e=z=0	b=j=z=0	e=j=z=0	b=e=j=z=0
order	80	82	80	82	78
	otherwise				
order	94				



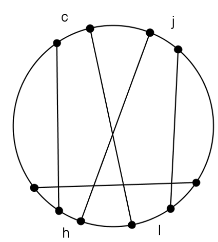
	b=0	h=0	ℓ=0	b=ℓ=0	b=h=0
order	88	88	94	86	84
	h=ℓ=0	h=z=0	b=h=ℓ=0	b=h=z=0	otherwise
order	86	82	82	80	96



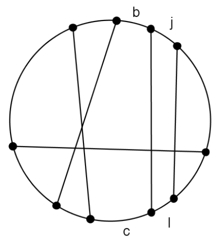
	e=0	h=0	e=h=0	h=ℓ=0	h=z=0
order	90	88	84	82	86
	e=z=0	h=e=z=0	h=e=ℓ=0	e=h=ℓ=z=0	otherwise
order	86	82	80	78	94



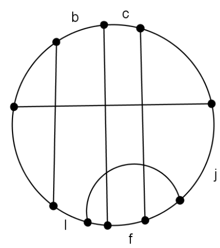
	b=0	h=0	ℓ=0	b=h=0
order	98	98	98	92
	h=ℓ=0	b=ℓ=0	h=e=0	h=j=0
order	92	90	90	90
	e=ℓ=0	h=j=e=ℓ=0	b=h=ℓ=e=0	b=h=ℓ=j=0
order	92	80	80	80
	b=h=e=0	b=h=j=0	b=h=ℓ=0	b=ℓ=e=0
order	84	86	86	84
	h=e=ℓ=0	h=j=ℓ=0	otherwise	
order	86	84	108	



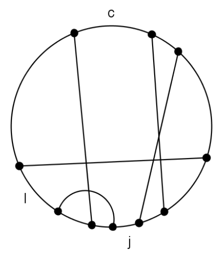
	c=0	c=h=0	c=j=0	c=ℓ=0
order	96	88	90	90
	c=j=ℓ=0	c=j=ℓ=h=0	otherwise	
order	84	80	104	



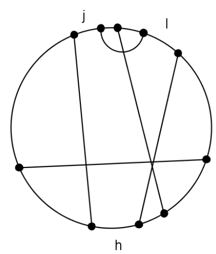
	b=0	b=c=0	b=j=0	b=ℓ=0
order	94	86	82	90
	j=0	b=j=c=0	otherwise	
order	94	78	102	



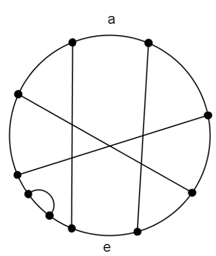
	j=0	b=0	b=c=0	b=c=j=0
order	90	96	80	76
	b=c=ℓ=0	b=f=0	b=f=ℓ=0	b=j=0
order	76	88	84	86
	b=j=f=0	b=j=f=ℓ=0	b=c=j=ℓ=0	b=j=ℓ=0
order	82	78	72	82
	b=ℓ=0	c=0	c=j=0	c=j=ℓ=0
order	90	90	82	78
	c=ℓ=0	f=0	f=ℓ=0	j=f=0
order	86	92	88	84
	j=f=ℓ=0	j=ℓ=0	ℓ=0	otherwise
order	80	86	96	102



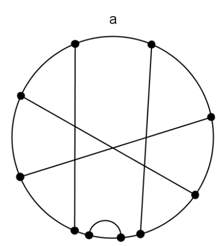
	c=0	c=j=0	c=j=ℓ=0	c=ℓ=0
order	94	90	86	90
	j=0	j=ℓ=0	ℓ=0	otherwise
order	98	90	94	102



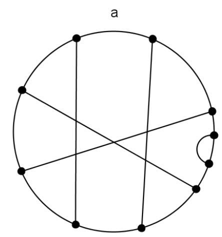
	h=0	h=j=0	h=ℓ=0	j=0
order	96	92	90	100
	j=ℓ=0	ℓ=0	otherwise	
order	92	96	104	



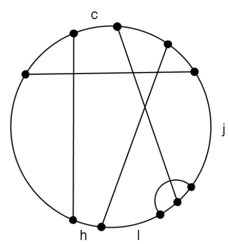
	a=0	e=0	otherwise
order	84	80	92



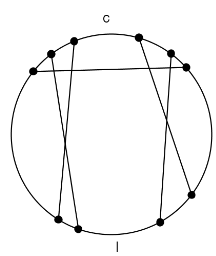
	a=0	otherwise
order	82	92



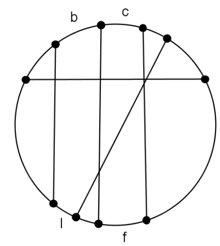
	a=0	otherwise
order	82	90



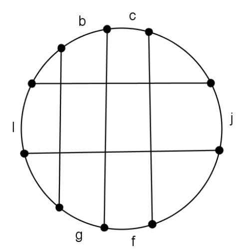
	c=0	h=0	c=h=0	c=j=0
order	94	94	86	88
	h=ℓ=0	c=ℓ=0	c=h=j=0	c=h=ℓ=0
order	84	88	82	78
	c=h=j=ℓ=0	otherwise		
order	74	102		



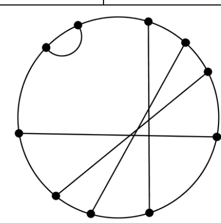
	$c = 0$	$c = \ell = 0$	otherwise
order	102	94	110



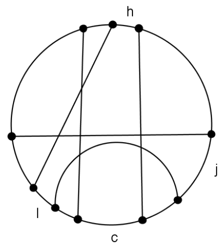
	b=0	c=0	f=0	ℓ=0
order	100	94	96	98
	b=c=0	b=f=0	b=ℓ=0	c=ℓ=0
order	84	92	92	88
	f=ℓ=0	b=c=ℓ=0	b=f=ℓ=0	otherwise
order	90	78	86	106



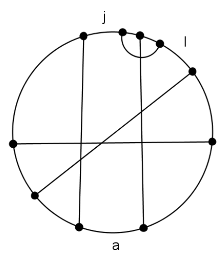
	b=0	ℓ=0	b=c=0	b=f=0	b=j=0
order	96	94	80	90	86
	b=ℓ=0	b=c=ℓ=0	b=f=ℓ=0	otherwise	
order	86	74	82	106	



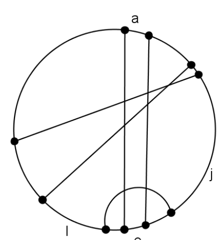
order=92



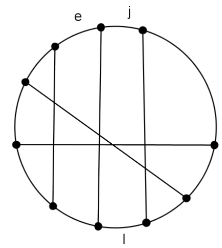
	c=0	h=0	j=0	ℓ=0
order	100	102	100	102
	h=c=0	h=j=0	h=ℓ=0	j=c=0
order	92	94	94	92
	j=ℓ=0	ℓ=c=0	h=j=c=0	h=j=ℓ=0
order	92	94	86	86
	h=ℓ=c=0	j=ℓ=c=0	h=j=ℓ=c=0	otherwise
order	86	86	80	112



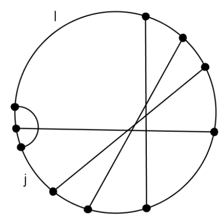
	a=0	j=0	ℓ=0	a=j=0	a=ℓ=0
order	94	100	100	90	90
	j=ℓ=0	a=j=ℓ=0	otherwise		
order	94	86	106		



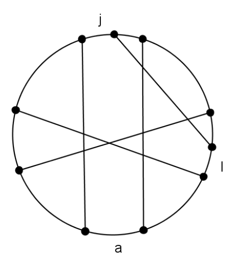
	a=0	e=0	ℓ=0	a=ℓ=0	e=ℓ=0
order	98	100	102	94 90	92
	ℓ=j=0	a=ℓ=j=0	e=ℓ=j=0	otherwise	
order	92	82	84	112	



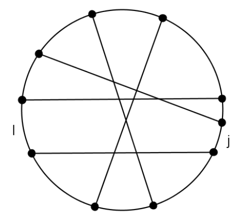
	e=0	e=j=0	e=ℓ=0	otherwise
order	98	82	92	108



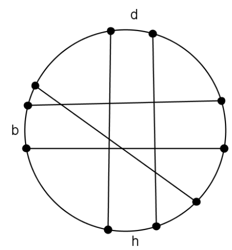
	ℓ=0	ℓ=j=0	otherwise
order	100	94	106



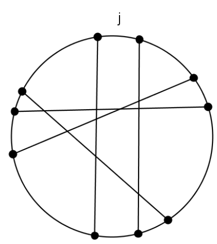
	a=0	j=0	a=ℓ=0	a=j=0	a=j=ℓ=0	otherwise
order	102	104	94	94	86	114



	j=0	ℓ=j=0	otherwise
order	102	92	112



	b=0	b=h=0	b=d=0	otherwise
order	100	90	88	114



	j=0	otherwise
order	98	112